4

VELOCITY ANALYSIS BY INSTANTANEOUS CENTER METHOD

Instantaneous centers (Centros)

Whenever two bodies have plane relative motion, at a particular instant that motion will be rotation about a center called *instantaneous center* or *center of rotation* or *rotopole* or *centro*. The instantaneous center of a moving body goes on changing from one instant to another. A centro can also be defined as a point common to two bodies having the same (zero) velocity in each or a point in one body about which another body actually turns.

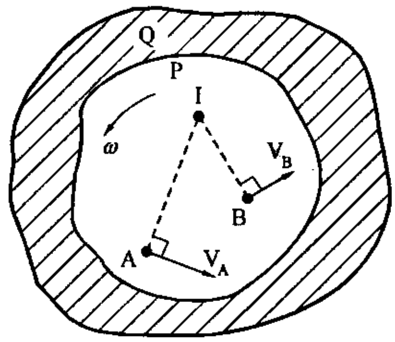

Fig. 4.1

Consider a body P having plane motion relative to a reference body Q as shown in fig. 4.1 The velocities V_A and V_B of two points A and B on P relative to Q are known. Lines drawn through A and B perpendicular to the directions of these velocities intersects at point I called *instant center*. The vector equations relating the velocity of point I to those of A and B are,

$$V_I = V_A + V_{IA} \quad \text{..... (1)}$$

$$V_I = V_B + V_{IB} \quad \text{..... (2)}$$

The velocity of I at this instant is zero. Therefore the above two equations becomes,

$$V_A = -V_{IA} \quad \text{and} \quad V_B = -V_{IB}$$

or $$V_A = V_{AI} \quad \text{and} \quad V_B = V_{BI} \quad \text{..... (3)}$$

Assume the body P is rotating at an angular velocity of ω, then

$$V_{AI} = \omega \times \text{IA} \quad \text{and} \quad V_{BI} = \omega \times \text{IB} \qquad \text{.....(4)}$$

$\therefore$ The equation (3) becomes

$$V_A = \omega \times \text{IA} \quad \text{and} \quad V_B = \omega \times \text{IB} \qquad \text{.....(5)}$$

or

$$\omega = \frac{V_B}{V_A} = \frac{\text{IB}}{\text{IA}} \qquad \text{.....(6)}$$

The magnitudes of the velocities of points on a rigid body having plane motion vary directly as the distances from the points to the instant centers. Thus if the instant center and the velocity of one point on a link are known, the velocity of any other point on the same link can be found.

The basic properties of an instant center are :

1. It is a point at which the two bodies having plane relative motion are relatively at rest.
2. It is the point about which the bodies are rotating relative to one another at the instant.

Aronhold Kennedy's theorem

The theorem states that the *instant centers for any three bodies having plane motion lie along the same straight line*. Let A, B and C be any three bodies having plane motion with respect to one another and *ba*, *ca*, and *bc* be the three centers as shown in fig. 4.2. *bc* is a point on either B or C. First consider *bc* as a point on B. Then it is moving relative to A about the centro *ba*, and its direction of motion is perpendicular to the line *ba-bc*. Next consider *bc* as a point on C. It is now moving relative to A about the centro *ca* and its direction of motion is perpendicular to the line *ca – bc*. But the point *bc* cannot have two different motions relative to A at the same time. Therefore, the perpendiculars to the lines *ba-bc* and *ca-bc* must coincide. This can occur only when *ba-bc-ca* is a straight line.

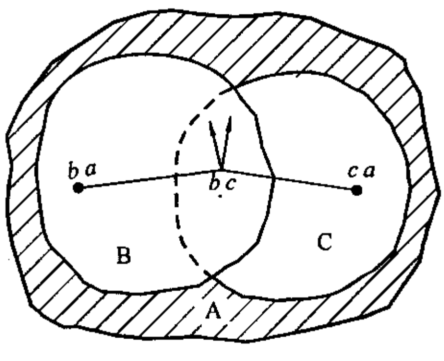

Fig. 4.2

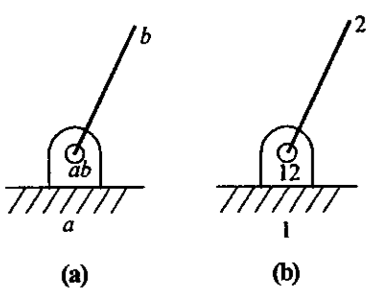

Fig. 4.3

Kennedy's theorem is very useful in locating instant centers in mechanism in cases where two centros for three links are known and the third has to be found.

Centro notation : There are a number of ways in which the centros are commonly designated. The notations of fig. 4.3 and are employed in this text.

Number of centros : For each pair of bodies having relative plane motion there is a centro. Therefore in a mechanism there are as many centros as there are possible pairs of links. The number of centros for a given number of links n can be obtained from the following expression.

$$\text{Number of centros} \quad N = \frac{n(n-1)}{2}$$

Velocity by centros

Linear velocity: Consider a four bar mechanism as shown in fig. 4.4. The centros are located and the linear velocity of the centro 23 is known for the mechanism. The centro 24 is a point common to links 2 and 4 and has the same velocity in each. The link 2 turns about centro 12 and the velocities of all points in 2 are proportional to their distances from 12. With 12 as center and 12-23 as radius, draw an arc to cut the line of centers at a. From a, draw vector ab of length equal to the linear velocity of the centro 23. Draw the line of proportion by joining 12 and b. Through the centro 24, draw a line parallel to ab to intersect the line of proportion at c. From similar triangles 12-a-b and 12-24-c.

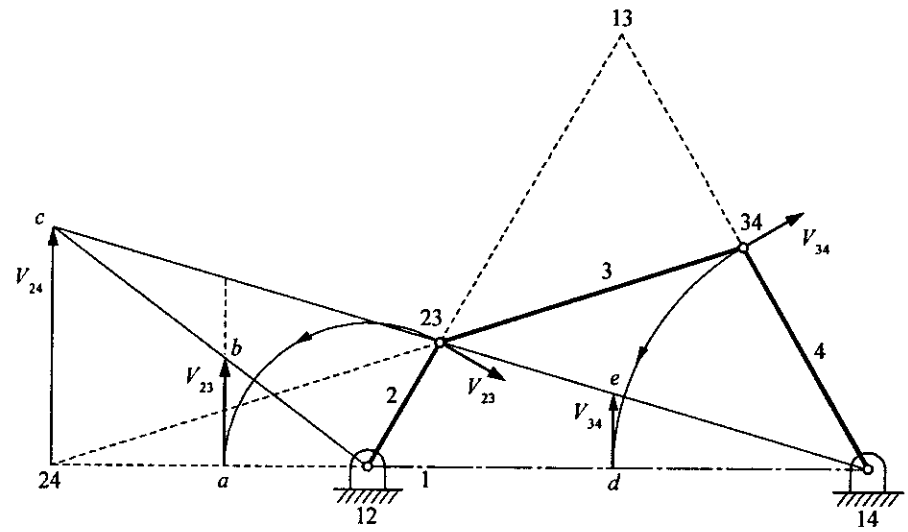

Fig. 4.4

$$\frac{V_{24}}{(12-24)} = \frac{V_{23}}{(12-23)}$$

$$\therefore \qquad V_{24} = V_{23}\,\frac{(12-24)}{(12-23)}$$

Now the velocity of 24, a point in link 4 is known. The velocity of 34, another point in 4 is desired. From similar triangles 14-*d*-*e* and 14-24-*c*.

$$\frac{V_{34}}{(14-34)} = \frac{V_{24}}{(14-24)}$$

$\therefore$ Linear velocity of the centro 34, $V_{34} = V_{24}\,\dfrac{(14-34)}{(14-24)}$

Angular velocity of the links : Referring to fig. 4.4 and considering the centro 24 to be in link 2,

$$V_{24} = \omega_2\,(12 - 24)$$

When 24 is considered to be in link 4,

$$V_{24} = \omega_4\,(14 - 24)$$

But the velocity of 24 is the same in each link.

$$\therefore \quad \omega_4\,(14 - 24) = \omega_2\,(12 - 24)$$

or $$\frac{\omega_4}{\omega_2} = \frac{12-24}{14-24}$$

The *angular velocity ratio theorm* states that the instantaneous angular velocities of two links are inverely proportional to the distances from their common centro to the centers about which they are turning or tending to turn.

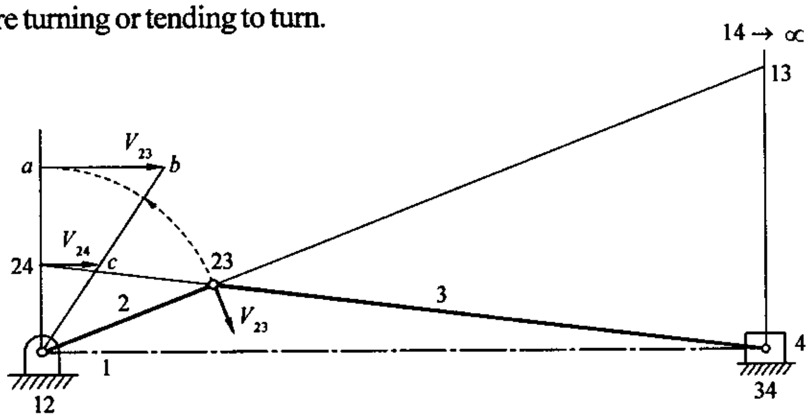

Fig. 4.5

Fig. 4.5 shows a slider crank mechanism with all centros located and the linear velocity of 23 is known. From similar triangles 12-*a*-*b* and 12-24-*c*,

$$\frac{V_{23}}{12-23} = \frac{V_{24}}{12-24}$$

$\therefore$ Linear velocity of the centro 24, $V_{24} = V_{23}\,\dfrac{(12-24)}{(12-23)}$

Since 24 and 34 are moving in the same direction, the velocity of $V_{34} = V_{24}$ which is the velocity of the slider.

Centros by inspection

The following rules are used in locating centros. These rules permit to find some primary centros by inspection. In any mechanism the primary centros must be located first, the rest being found by construction.

1. If two links are connected by a pivoted joint, the pivot itself is a centro for the two links (refer fig. 4.6a)
2. In a sliding block, the centro lies at infinity in a direction perpendicular to the path of motion of the slider. (refer fig. 4.6b)

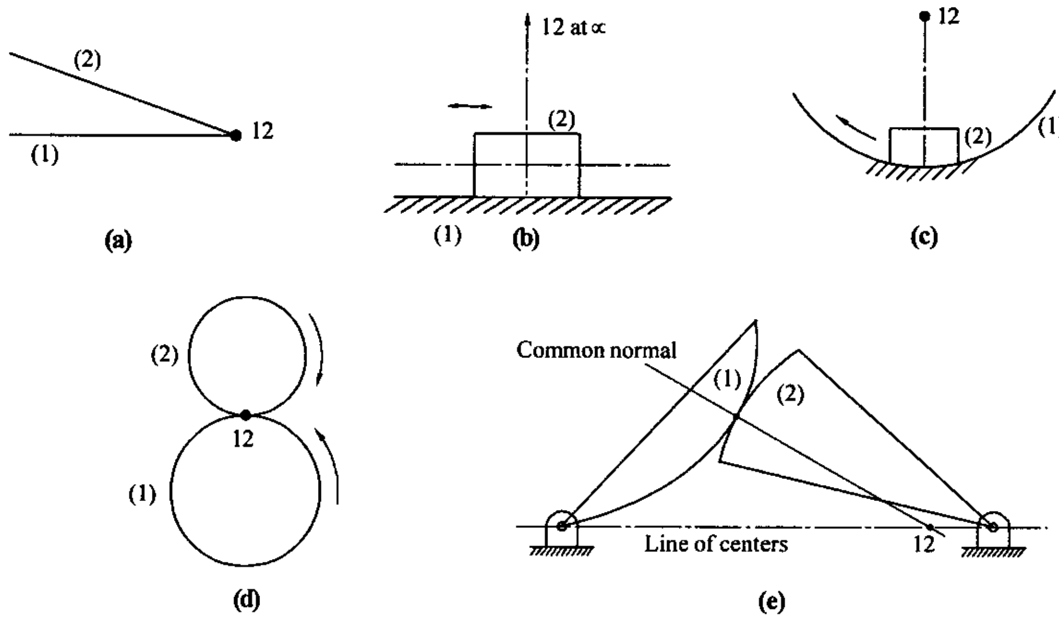

Fig 4.6

3. If a link is sliding over a curved element, the center of curvature is the centro. (refer fig. 4.6c)
4. If two links are having rolling contact, the centro lies at the point of contact. (refer fig. 4.6d)
5. If two links have sliding contact, their centro lies where the common normal to the surfaces at the point of contact intersects the line of centers (refer fig. 4.6e).

Circle diagram

A circle diagram or *instant center polygon* is used to keep track of those instant centers which have already been found and for developing a strategy to find any instant center not yet located. The polygon must have as many sides as the mechanism has links. There will be just as many sides and diagonals in the polygon as there are instant centers in the mechanism, and each instant center will be represented by a side or a diagonal of the polygon.

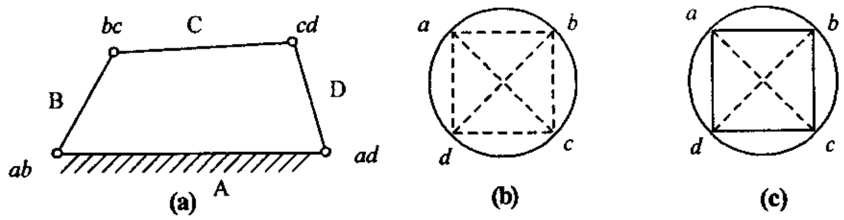

Fig. 4.7

Consider a four bar mechanism as shown in fig 4.7a. To construct a circle diagram, a small circle of arbitrary radius is drawn first. The circumference of the circle is then divided into as many equal spaces as there are links in the mechanism. Since there are four links, the instant center polygon will be a square. Light dashed lines are then drawn from each point to every other point so that the diagram appears as shown in fig. 4.7b. Next, those lines representing instant centers whose locations are known are drawn in heavy. Fig. 4.7c then shows clearly that the locations of instantaneous centers *ac* and *bd* are not yet known. From Kennedy's theorem, the three centers represented by the three sides of any of the triangles must lie on a straight line. By using Kennedy's theorem it is easy to find the other instant centers.

Example 4.1

Find the instant centers for a four bar mechanism shown in fig. 4.8.

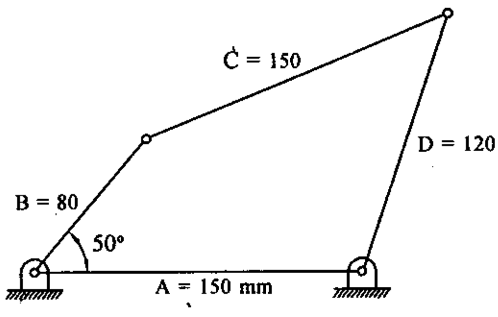

Fig. 4.8

Solution :

Number of links $n = 4$

Number of instant centers $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Draw the configuration diagram for the given mechanism with some suitable scale. Since there are four links, locate the four corners of the instant center polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corner.

Locate the primary instant centers on the configuration diagram. The four pin joints *ab*, *bc*, *cd* and *ad* are the four primary instant centers. Draw solid lines *ab*, *bc*, *cd* and *ad* on the circle diagram to represent the primary instant centers. (refer fig. 4.9*a*).

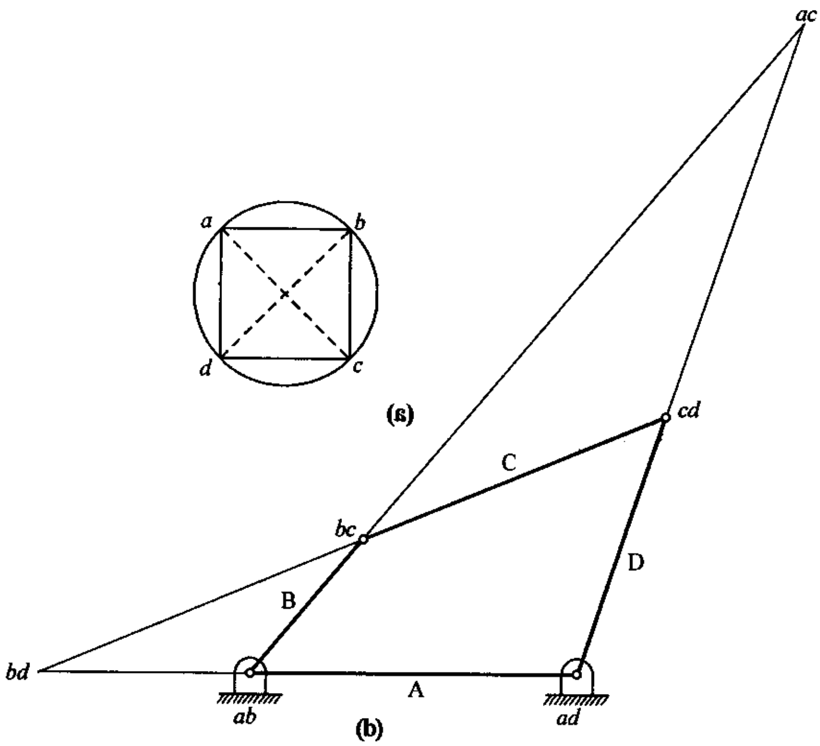

Fig. 4.9

In the circle diagram, there are two triangles *acd* and *abc* that share the unknown side *ac*. According to the triangle *acd*, centros *ac*, *ad* and *cd* must lie in a straight line and since *cd* and *ad* are already located, they establish a line that contains *ac*. Draw a line through *ad* - *cd*. According to triangle *abc*, centros *ab*, *bc* and *ac* must be on a straight line, and since *ab* and *bc* are already located, they establish a second line that contains *ac*. Draw a line through *ab* - *bc*. The inter section of these two lines locates centro *ac* as shown in fig. 4.9*b*.

Join the corners *a* and *c* of the centro polygon by solid line to represent that the centro *ac* is located.

Similarly, the two triangles *abd* and *bcd* that share the unknown side *bd*. Triangle *abd* contains the known centros *ab* and *ad* and thus establishes one line containing the centro *bd*. Triangle *bcd* contains the known centros *bc* and *cd* and thus establishes a second line containing the centro *bd*. The intersection of these two lines locates the centro *bd* as shown. Join the corners *b* and *d* of the centro polygon by solid line. If all the lines of the circle diagram have become solid, it indicates that all centros are determined.

Example 4.2

Locate the instant centers for the four bar mechanism shown in fig. 4.10

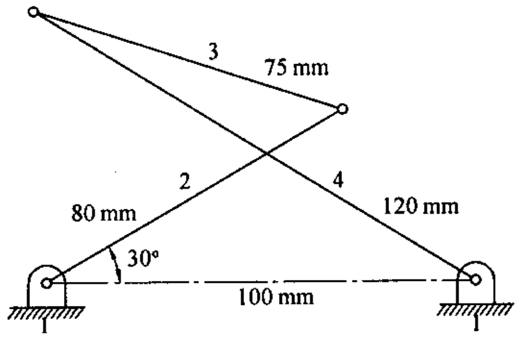

Fig. 4.10

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Since there are four links, locate the four corners of the regular polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corner.

Draw the configuration diagram for the given four bar mechanism and locate the primary centros. The four pin joints 12, 23, 34 and 14 are the four primary centros. Draw solid lines 12, 23, 34 and 14 on the circle diagram (refer fig. 4.11*a*)

In the circle diagram, the side 13 is common to two triangle, 123 and 134. Locate the centro 13 which lies at the inter section of lines 12-23 and 14-34 (refer fig. 4.11*b*). Join the corner 1 and 3 of the centro polygon by solid line to represent that the centro 13 is located.

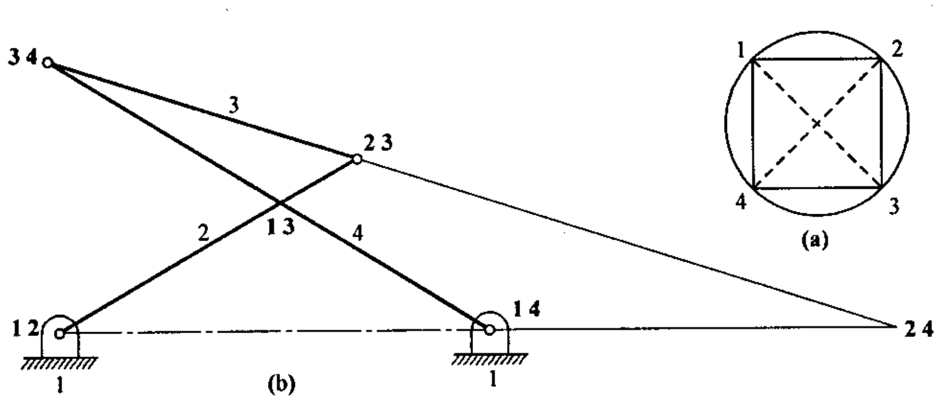

Fig. 4.11

In the circle diagram, the side 24 is common to two triangle 124 and 234. Locate the centro 24, which lies at the intersection of lines 12-14 and 23-34. (refer fig. 4.11*b*). Join the corners 2 and 4 of the centro polygon by solid line. As all the sides and diagonals of the centro polygon are solid, it indicates that all centros have been determined.

Example 4.3

In a four bar mechanism shown in fig. 4.12, the link 2 is driven at 36 rad/s. Find the velocity of link 3 and the velocity of point B by instantaneous center method. Link $O_2A = 50$ mm, $O_4B = AB = 200$ mm, distance $O_2O_4 = 175$ mm.

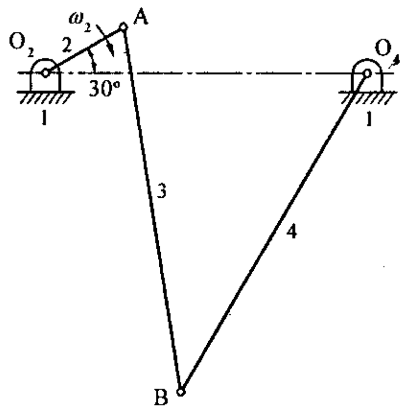

Fig. 4.12

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Since there are four links, locate the four corners of the regular polygon on the circumference of a circle and lable the corners. Draw thin dashed lines from each corner to every other corner.

Draw the configuration diagram for the given four bar mechanism and locate the primary centros. The four pin joints 12, 23, 34 and 14 are the primary centros. Draw solid lines in the centro polygon to represent the centros, whose locations are known (refer fig. 4.13a). Locate the two remaining centros by using Kennedy's theorm.

1. From the centro polygon, the side 13 is common between two triangles 123 and 134. Locate the centro 13 which lies at the intersection of lines 12-23 and 14-34. Join corners 1 and 3 of the centro polygon by solid line to represent that the centro 13 is located.
2. Similarly, the side 24 is common between two triangles 124 and 234. Locate the centro 24 which lies at the intersection of lines 12-14 and 23-34. Join the corners 2 and 4 of the centro polygon by solid line.

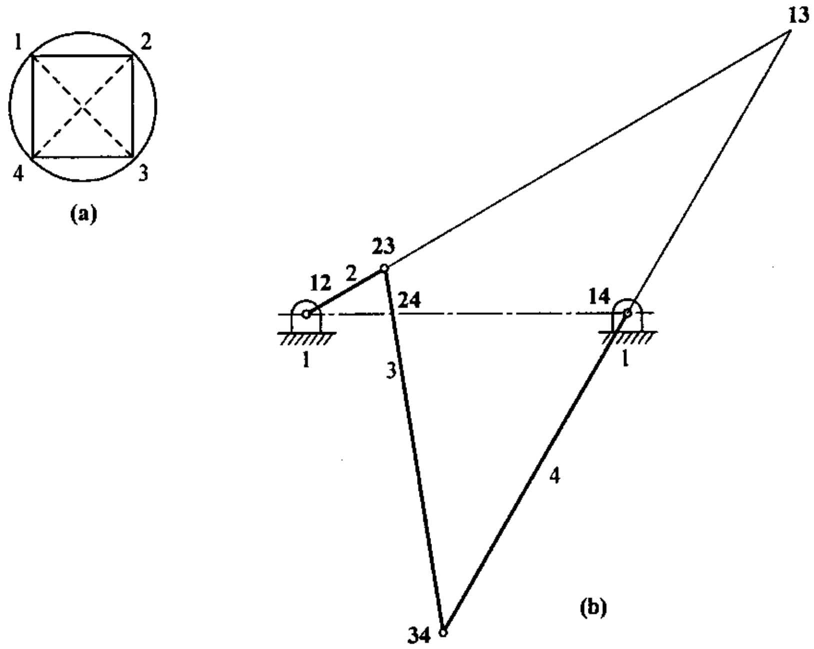

Fig. 4.13

Angular velocity of link 2, $\omega_2 = 36$ rad/s

Velocity of A $= \omega_2 \times O_1A = 36 \times 50 = 1800$ mm/s

Consideer the link 3 and its centro 13 shown in fig. 4.13b.

On measurement, distance IA = 13-23 = 254.96

and IB = 13-34 = 376.52

We know that $\frac{V_A}{IA} = \frac{V_B}{IB}$

$$\text{Velocity of B} = V_A \times \frac{IB}{IA}$$

$$= 1800 \times \frac{376.52}{254.96} = 2658.2 \text{ mm/s}$$

Angular velocity of link 3, $\omega_3 = \frac{V_A}{IA} = \frac{V_B}{IB} = \frac{1800}{254.96} = 7.06$ rad/s

Angular velocity of link 4, $\omega_4 = \frac{V_B}{O_4B} = \frac{2658.2}{200} = 13.29$ rad/s

Example 4.4

Locate all the instant centers for the slider crank mechanism shown in fig. 4.14. The length of various links are: OA = 150 mm, AB = 480 mm and OB = 600 mm.

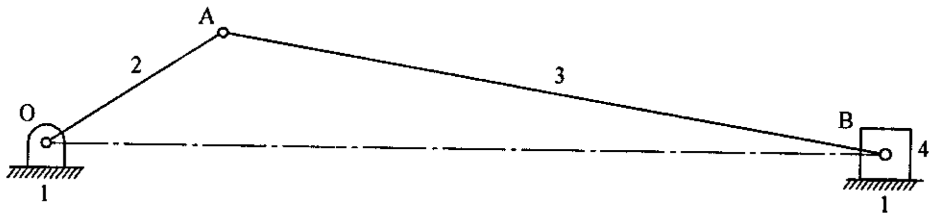

Fig. 4.14

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Since there are four links, locate the four corners of the regular polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corner.

Draw the configuration diagram for the given slider crank mechanism and locate the primary centros as shown in fig. 4.15*b*. The three pin joints 12, 23 and 34 are the three primary centros. The fourth primary centro 14 lies at infinity in a direction perpendicular to the path of motion of the slider. Draw solid lines 12, 23, 34 and 14 on the circle diagram to represent the known primary centros. (refer fig. 4.15*a*).

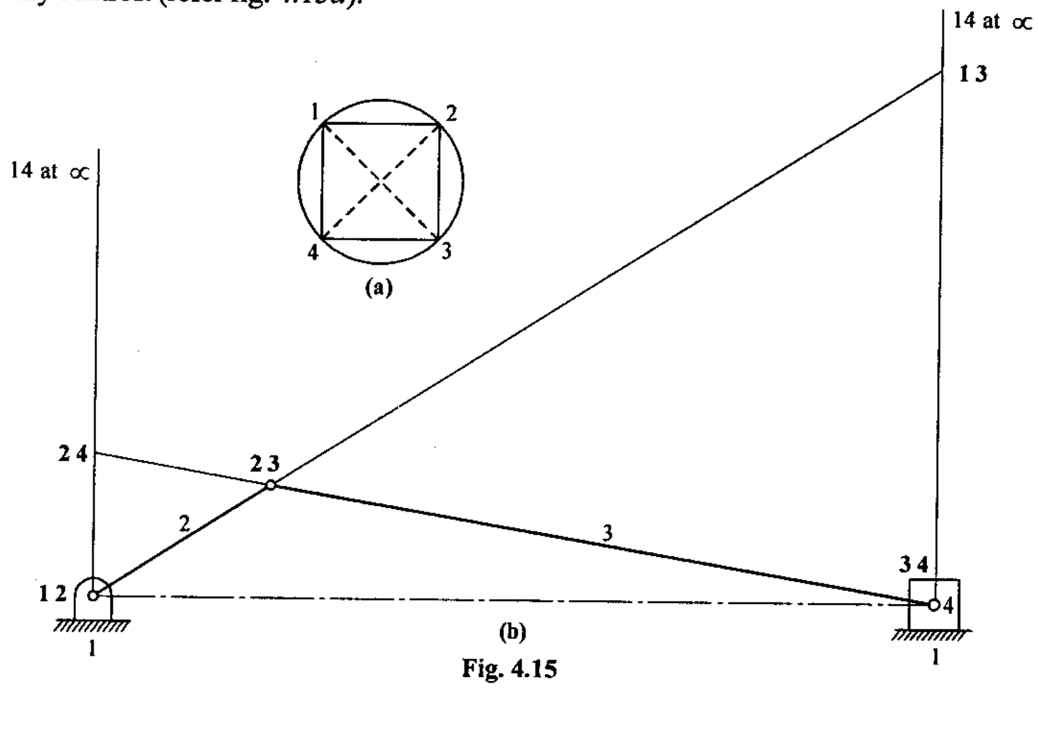

Fig. 4.15

In the circle diagram, the side 13 is common to two triangles 123 and 134. From triangle 123, centros 12 and 23 are already located, so they establish one line containing the centro 13. From triangle 134, centros 34 and 14 are already located, so they establish the other required line to locate 13. Centro 13 is thus located at the intersection of lines 12 - 23 and 14 - 34. The line 14 - 34 is drawn from 34 and normal to the direction of motion of the slider. Join the corners 1 and 3 of the centro polygon by solid line which indicates that the centro 13 is determined.

Referring to the circle diagram, the side 24 is common to two triangles 124 and 234. From triangle 234, centros 23 and 34 are already located, so they establish one line containing the centro 24. From triangle 124, centros 12 and 14 are already located, so they establish the other required line. In this case, to draw a line through centros 12 and 14, it is necessary to move 14 to where it will pass through 12. This utilizes the concept that parallel lines meet at infinity. The intersection of the two lines 23-34 and 12-14 is the centro 24. Join the corners 2 and 4 of the centro polygon by solid line. As all the sides and the diagonals of the centro polygon are solid, it indicates that all centros have been determined.

Example 4.5

Locate the instant centers for the double slider mechanism shown in fig. 4.16a.

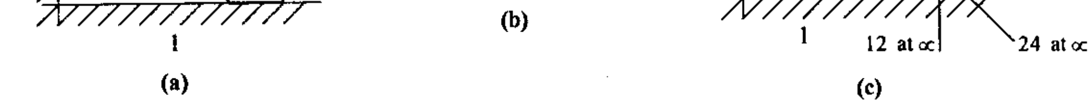

Fig. 4.16

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Since there are four links, locate the four corners of the regular polygon on the circumference of a circle and lable the corners. Draw thin dashed line from each corner to every other corner.

Draw the configuration diagram for the given mechanism and locate the primary centros as shown in fig. 4.16c. The two pin joints 23 and 34 are the two primary centros. The third primary centro 12 lies at infinity in a direction perpendicular to the path of motion of slider 2 and the fourth primary centro 14 lies at infinity in a direction perpendicular to path of motion of slider 4. Draw solid lines 12, 23, 34 and 14 on the circle diagram to represent the known primary centros (refer fig. 4.16b). Locate the 2 remaining centros by using Kennedy's theorm.

1. From the centro polygon, the side 13 is common between the two triangles 123 and 134. Locate the centro 13 which lies at the intersection of the lines 12-23 and 14-34. Join the corners 1 and 3 of the centro polygon to represent the centro 13 is located.
2. Similarly the side 24 is common between the two triangles 124 and 234. Locate the centro 24 which lies at the intersection of lines 12-14 and 23-34. Join the corners 1 and 4 of the centro polygon by solid line.

Example 4.6

Fig. 4.17 shows a Scotch-yoke mechanism. It is driven by the crank 2 at 1 rad/s. Find the velocity of the cross-head(link 4) by instantaneous center method. Length of the crank is 50 mm.

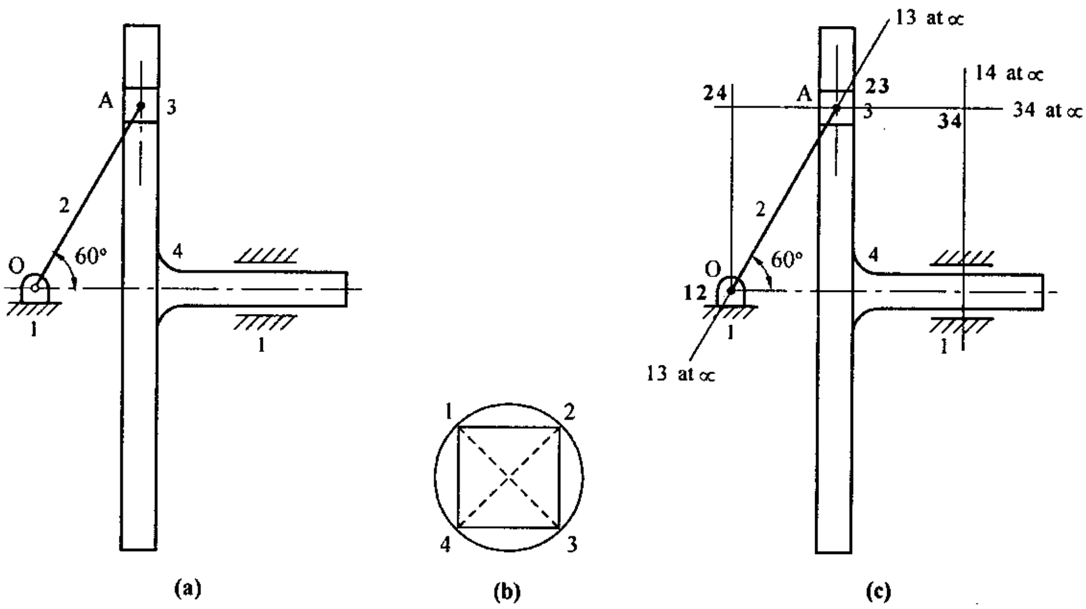

Fig. 4.17

Solution:

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Since there are four links, locate the four corners of the regular polygon on the circumference of a circle and lable the corners. Draw thin dashed lines from each corner to every other corners.

Draw the configuration diagram for the given Scotch-yoke mechanism and locate the primary centros. Locate the 4 primary centros by inspection. There are 2 pin joints and two sliding pairs. Centros 12 and 23 are the centers of the pin joints. Centro 34 is at infinity in a direction perpendicular to the motion of block 3 and the centro 14 is at infinity in a direction perpendicular to the motion of the cross-head 4.

Locate the remaining 2 unknown centros by Kennedy's theorem.

Unknown side (Centro)	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	134	12-23	14-34	13
24	124	234	12-14 Move the vertical line 14 to 12	23-34	24

Angular velocity of the crank $\omega_2 = 1$ rad/s

Velocity of A, $V_A = \omega_2 \times OA = 1 \times 50 = 50$ mm/s

Consider the links 2 and 4 and the common centro 24,

$$\frac{V_A}{12-23} = \frac{V_4}{12-24}$$

$\therefore$ On measurement, distance (12-23) = 50 mm and (12-24) = 43.3 mm

$$\therefore \text{ Velocity of the cross-head } \quad V_4 = V_A \frac{(12\text{-}24)}{(12\text{-}23)}$$

$$= 50 \times \frac{43.3}{50} = 43.3 \text{ mm/s}$$

Example 4.7

Locate the centros for the three - link cam mechanism shown in fig. 4.18.

Solution :

Number of links $n = 3$

Number of instant centers $N = \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$

Since there are only three links, the circle diagram is unnecessary. Draw the configuration diagram for the given mechanism and locate the primary centros by inspection. The centro 12 is where the cam pivots relative to the frame. Since the follower 3 has a sliding motion relative to the frame, the centro 13 is at infinity in a direction perpendicular to the direction of motion of the

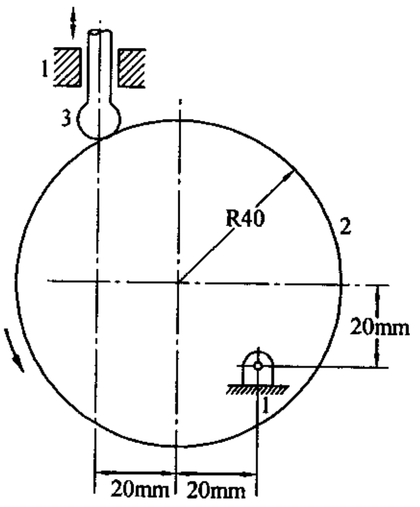

Fig. 4.18

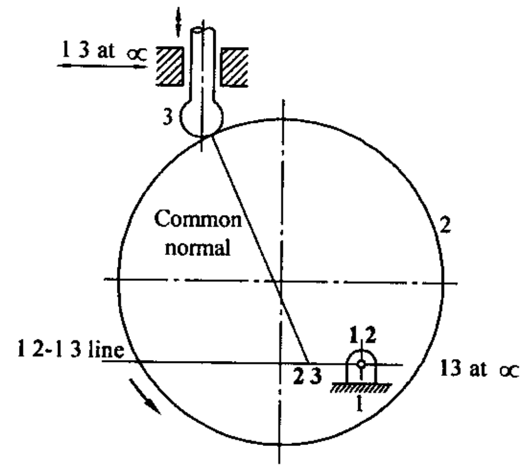

Fig. 4.19

follower. The links 2 and 3 have sliding contact, the centro 23 must lie on the common normal to the surface at the point of contact. By kennedy's theorem, centros 12, 13 and 23 must lie in a straight line. Draw line 12 - 13 from 12 and in a direction perpendicular to the direction of motion of the follower. The intersection of the line 12-13 and the common normal is the centro 23 (fig. 4.19).

Example 4.8

Locate the instant centers for the four link cam mechanism shown in fig. 4.20.

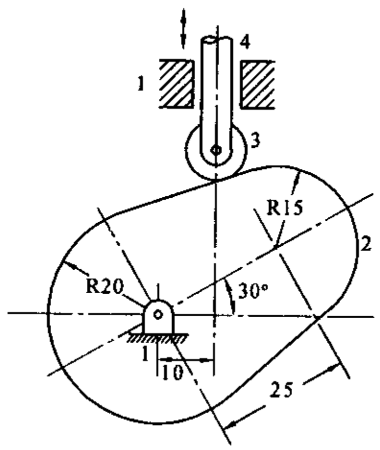

Fig. 2.20

Solution :

Number of links $n = 4$

Number of instant centers $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Draw the configuration diagram for the given mechanism. Since there are four links, locate the four corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corner.

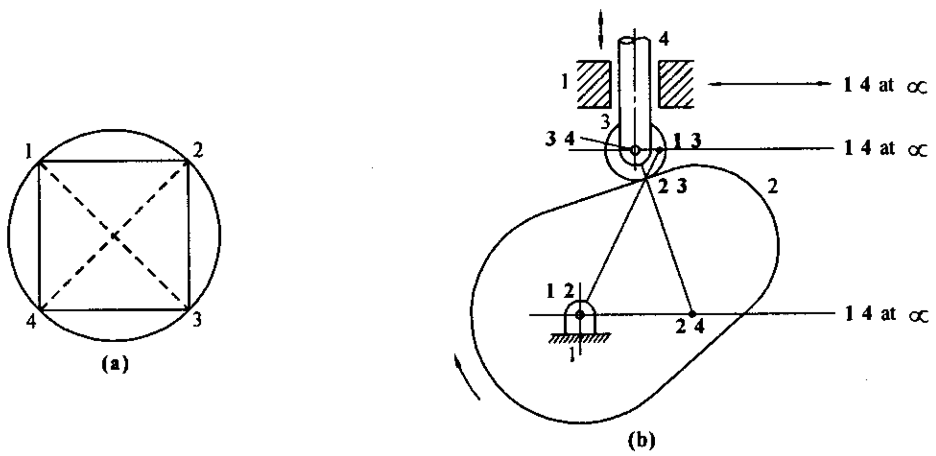

Fig. 4.21

Locate the four primary centros. The two primary centros are the two pin joints, i.e., the cam to frame pivot 12, and the roller pivot 34. There will be pure rolling contact between the roller 3 and the cam 2, so the centro 23 is at their point of contact. The fourth primary centro 14 is at infinity in a direction of motion of the translating rod 4 relative to the frame 1. Draw solid lines 12, 23, 34 and 14 on the circle diagram to represent the known centros (fig. 4.21a).

In the circle diagram, the side 13 is common to two triangles 123 and 134. From triangle 123, centros 12 and 23 establish a line containing the centro 13. From triangle 134, centros 14 and 34 establish a second line containing the centro 13. In this case, draw line 14-34 from 34 and in a direction normal to the direction of motion of the translating rod. The intersection of the line 12-23 and 14-34 is the centro 13. Join the corners 1 and 3 of the centro polygon by solid line.

Referring to the circle diagram, the side 24 is common to two triangles 124 and 234. From triangle 124, centros 12, and 14 establish a line containing the centro 24. In this case, draw a line 12-14 from 12 and in a direction normal to the direction of motion of the translating rod. From triangle 234, centros 23 and 34 establish another line containing the centro 24. The intersection of the line 12-14 and 23-34 is the centro 24. Join the corners 2 and 4 of the centro polygon by solid line. As all the sides and diagonals of the centro polygon are solid, it indicates that all centros have been determined.

Example 4.9

In a four bar chain mechanism ABCD, the link AD is fixed, which is 120 mm long. The links AB, BC and CD are 60 mm, 80 mm, and 80 mm long respectively. At certain instant the crank AB makes an angle of 60° with the fixed link AD. If the crank AB rotates at uniform speed of 10 rpm clockwise, determine the angular velocity of the links BC and CD by instantaneous center method.

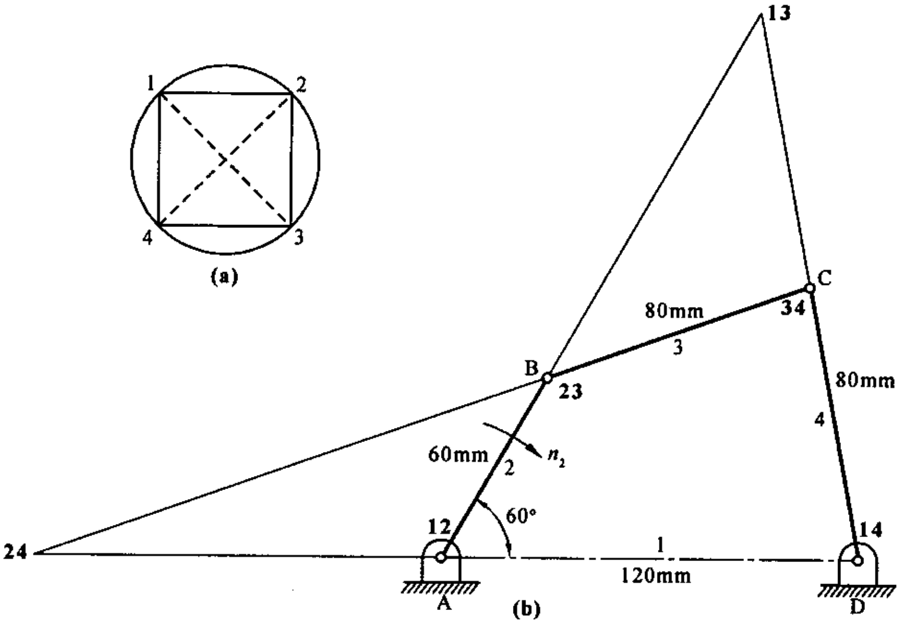

Fig. 4.22

Solution :

Number of links $\quad n = 4$

Number of centros $\quad N = \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 6$

Draw the configuration diagram of the given mechanism with suitable scale. Since there are 4 links, locate the four corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

Locate the 4 primary centres by inspection. The four pin joints 12, 23, 34 and 14 are the primary centros. Draw solid lines in the centro polygon to represent the centros, whose locations are known. (refer fig. 4.22a).

Locate the two remaining centros by using kennedy's theorem.

1. From the centro polygon, the side 13 is common between the two triangles 123 and 134. Locate the centro 13 which lies at the intersection of the lines 12-23 and 14-34. Join the centro polygon corners 1 and 3 by solid line to represent that the centro 13 is located.
2. Similarly, the side 24 is common between the two triangles 124 and 234. Locate the centro 24 which lies at the intersection of the lines 12-14 and 23-34. Join the corners 2 and 4 of the centro polygon by solid line. If all the lines of the centro polygon have become solid, it indicates that all centros are determined.

Speed of the crank AB, $n_2 = 10$ rpm

Angular velocity of the crank AB, $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 10}{60} = 1.047$ rad/s

Velocity of B, $V_B = \omega_2 \times AB = 1.042 \times 60 = 62.832$ mm/s

Consider the link 3 and its centro 13, on measurement (fig. 2.32*b*)

Distance $IB = 13 - 23 = 121$ mm

and $IC = 13 - 34 = 79$ mm

We know that, $\dfrac{V_B}{IB} = \dfrac{V_c}{IC}$

$\therefore$ Velocity of C, $V_C = V_B \times \dfrac{IC}{IB} = 62.832 \times \dfrac{79}{121} = 41.0225$ mm/s

Angular velocity of link 3, $\omega_3 = \dfrac{V_B}{IB} = \dfrac{V_c}{IC}$

$$= \frac{62.832}{121} = 0.5193 \text{ rad/s}$$

Angular velocity of link 4, $\omega_4 = \dfrac{V_c}{CD} = \dfrac{41.0225}{80} = 0.5128$ rad/s

OR Ratio of angular velocities $\dfrac{\omega_3}{\omega_2} = \dfrac{23-12}{23-13}$ (consider links 2, 3 and its centro 23)

On measurement (fig. 4.22*b*),

Distance $23 - 12 = 60$ mm and $23 - 13 = 121$ mm

Angular velocity of link 3, $\omega_3 = \dfrac{23-12}{23-13} \times \omega_2 = \dfrac{60}{121} \times 1.0472 = 0.5193$ rad/s

Similarly, $\dfrac{\omega_4}{\omega_2} = \dfrac{24-12}{24-14}$ (consider links 2, 4 and its centro 24)

On measurement (fig. 4.22*b*),

Distance $24 - 12 = 117$ mm and $24 - 14 = 239$ mm

$\therefore$ Angular velocity of link 4, $\omega_4 = \dfrac{24-12}{24-14} \times \omega_2 = \dfrac{117}{239} \times 1.0472 = 0.5126$ rad/s

---

Example 4.10

In the four-bar mechanism shown in fig. 4.23, the link 2 is rotating at an angular velocity of 10 rad/s. The length of various links are: $O_2A = 100$ mm, $O_4B = 100$ mm, AB = 200 mm, AC = 150 mm and BC = 100 mm. Locate all the instantaneous centers of the mechanism and find : (i) Angular velocities of links 3 and 4, and (ii) Linear velocities of points A, B, C and D.

The point D is on link 4 and is located at a distance of 75 mm from B.

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

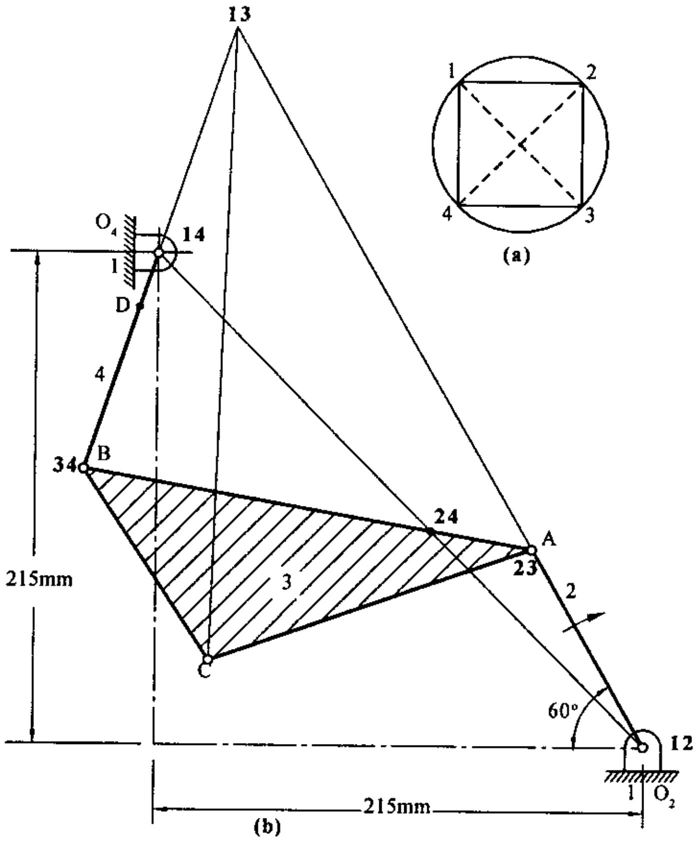

Fig. 4.23

Draw the configuration diagram of the given mechanism with suitable scale. Since there are 4 links, locate the four corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

Locate the 4 primary centres by inspection. The four pin joints 12, 23, 34 and 14 are the primary centros. Draw solid lines in the centro polygon to represent the centros, whose locations are known. (refer fig. 4.23*a*).

Locate the two remaining centros by using kennedy's theorem.

1. From the centro polygon, the side 13 is common between the two triangles 123 and 134. Locate the centro 13 which lies at the intersection of the lines 12-23 and 14-34. Join the centro polygon corners 1 and 3 by solid line to represent that the centro 13 is located.

2. Similarly, the side 24 is common between the two triangles 124 and 234. Locate the centro 24 which lies at the intersection of the lines 12-14 and 23-34. Join the corners 2 and 4 of the centro polygon by solid line. If all the lines of the centro polygon have become solid, it indicates that all centros are determined.

Angular velocity of link 2, $\omega_2 = 10$ rad/s

Consider links 2, 3 and its centro 23,

Ratio of angular velocities $\dfrac{\omega_3}{\omega_2} = \dfrac{23-12}{23-13}$

On measurement (fig. 4.23*b*),

Distance 23-12 = 100 mm and 23-13 = 262.8 mm

∴ Angular velocity of link 3, $\omega_3 = \dfrac{23-12}{23-13} \times \omega_2 = \dfrac{100}{262.8} \times 10 = 3.805$ rad/s

Similarly for links 2 and 4, $\dfrac{\omega_4}{\omega_2} = \dfrac{24-12}{24-14}$

On measurement (fig. 4.23*b*),

Distance 24-12 = 133 mm and 24-14 = 171 mm

∴ Angular velocity of link 4, $\omega_4 = \dfrac{24-12}{24-14} \times \omega_2 = \dfrac{133}{171} \times 10 = 7.78$ rad/s

Velocity of A, $V_A = \omega_2 \times O_2A = 10 \times 1000 = 1000$ mm/s

Consider the links 1, 3 and its centro 13,

On measurement (fig. 4.23*b*),

Distance IA = 13-23 = 262.8 mm

IB = 13-34 = 204.7 mm

and IC = 13-C = 277 mm

We known that $\dfrac{V_A}{IA} = \dfrac{V_B}{IB} = \dfrac{V_c}{IC}$

Velocity of B, $V_B = \dfrac{IB}{IA} \times V_A = \dfrac{204.7}{262.8} \times 1000 = 778.92$ mm/s

Velocity of C, $V_C = \dfrac{IC}{IA} \times V_A = \dfrac{277}{262.8} \times 1000 = 1054.03$ mm/s

Consider the link 1, 4 and its centro 14,

Distance IB = 14 – 34 = O_4B = 100 mm

Distance ID = 14 – D = O_4B – BD = 100 – 80 = 20 mm

We know that $\frac{V_B}{IB} = \frac{V_A}{ID}$

$\therefore$ Velocity of D, $V_D = \frac{ID}{IB} \times V_B = \frac{20}{100} \times 778.92 = 155.784$ mm/s

---

Example 4.11

The mechanism shown in fig. 4.24 has the following dimensions: OA = 100 mm, AB = 280 mm, BC = 240 mm, and CD = 120 mm. The center of gravity G of the link AB is located at a distance of 120 mm from A. Determine the velocity of G, D and the angular velocity of link AB and the bell crank lever BCD. The crank OA rotates uniformly at 30 rad/s.

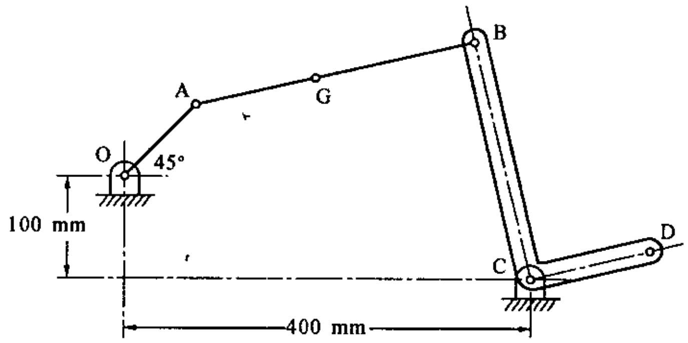

Fig. 4.24

Solution :

Number of links $n = 4$

Number of centros $N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

Draw the configuration diagram of the given mechanism with suitable scale. Since there are 4 links, locate the four corners of the centro polygon on the circumference of a circle of arbitrary radius and lable the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

Locate the 4 centros by inspection. The centros 12, 23, 34 and 14 are the centers of the pin joints. Draw solid lines in the centro polygon representing the centros whose locations are known (fig. 4.25a)

Locate the two remaining centros by using Kennedy's theorm as shown in the table.

Angular velocity of the crank OA, $\omega_2 = 30$ rad/s

Velocity of A $V_A = \omega_2 \times OA = 30 \times 100 = 3000$ mm/s

Unknown side (Centro)	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	134	12-23	14-34	13
24	124	234	12-14	23-34	24

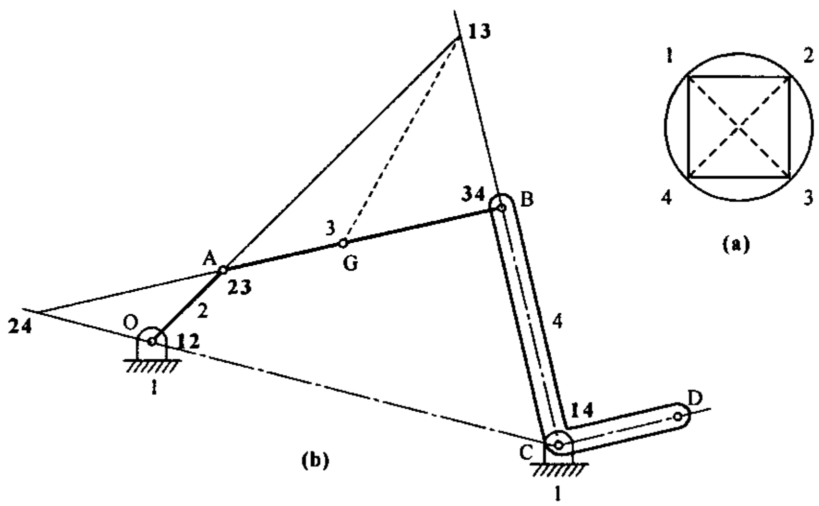

Fig. 4.25

Points A, B and G lies on link 3. Consider the link 1, 3 and its centro 13,

$$\frac{V_A}{13\text{-}23} = \frac{V_B}{13\text{-}34} = \frac{V_G}{13\text{-}G} = \omega_3$$

On measurement, distance (13-23) = 328.12mm, (13-34) = 174.3mm, (13-G) = 235.24mm

$\therefore$ Velocity of B, $$V_B = V_A \frac{(13\text{-}34)}{(13\text{-}23)} = 3000 \times \frac{174.3}{328.12} = 1593.6 \text{ mm/s}$$

Velocity of G, $$V_G = \frac{V_A(13\text{-}G)}{(13\text{-}23)} = 3000 \times \frac{235.24}{328.12} = 2150.8 \text{ mm/s}$$

Angular velocity of link 3, $$\omega_3 = \frac{V_A}{13\text{-}23} = \frac{3000}{328.12} = 9.14 \text{ rad/s}$$

Angular velocity of link 4, $$\omega_4 = \frac{V_B}{CB} = \frac{1593.6}{240} = 6.64 \text{ rad/s}$$

$\therefore$ Velocity of D, $$V_D = \omega_4 \times CD = 6.64 \times 120 = 796.8 \text{ mm/s}$$

Example 4.12

Fig. 4.26 shows a mechanism for a wrapping machine. The crank O_1A is a driving crank rotating at a uniform speed of 50 rad/s. The dimensions of the mechanism are: $O_1A = 80$ mm, AB = 650 mm, BC = 200 mm, $O_3B = 180$ mm, $O_2E = 350$ mm, $O_2D = 175$ mm and CD = 125 mm. Locate all the instantaneous center and find the velocity of the point E on the bell crank.

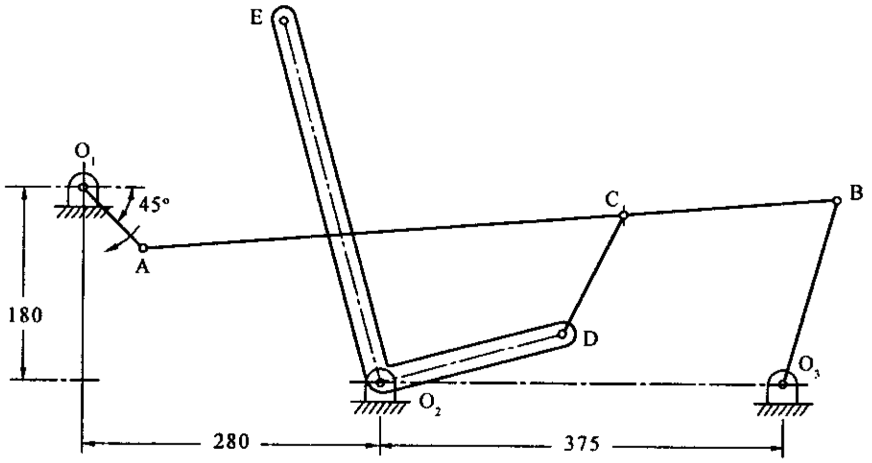

Fig. 4.26

Solution :

Number of links $n = 6$

Number of centros $N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$

Draw the configuration diagram for the given mechanism with suitable scale. Since there are 6 links, locate the six corners of the centro polygon on the circumference of a circle and lable the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

Locate the 7 primary centros by inspection. There are 7 pin joints. Centros 12, 14, 16, 23, 35, 34 and 56 are the centers of pin joints. Draw solid lines in the centro polygon representing centros whose locations are known (fig. 4.27a)

Locate the remaining 8 centros by Kennedy's theorm as shown in the table.

Angular velocity of link 2, $\omega_2 = 50$ rad/s

Velocity of A, $V_A = \omega_2 \times O_1A = 50 \times 80 = 4000$ mm/s

Consider the links 1, 3 and its centro 13,

$$\frac{V_A}{(13\text{-}23)} = \frac{V_C}{(13\text{-}35)}$$

On measurement, distance (13-23) = 696.42 mm, (13-35) = 528.61 mm

Unknown side (centro)	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	134	12-23	14-34	13
15	156	135	16-56	13-35	15
24	124	234	12-14	23-34	24
25	125	235	12-15	23-35	25
26	126	256	12-16	25-56	26
36	136	356	13-16	35-56	36
45	145	245	14-15	24-25	45
46	146	456	14-16	45-56	46

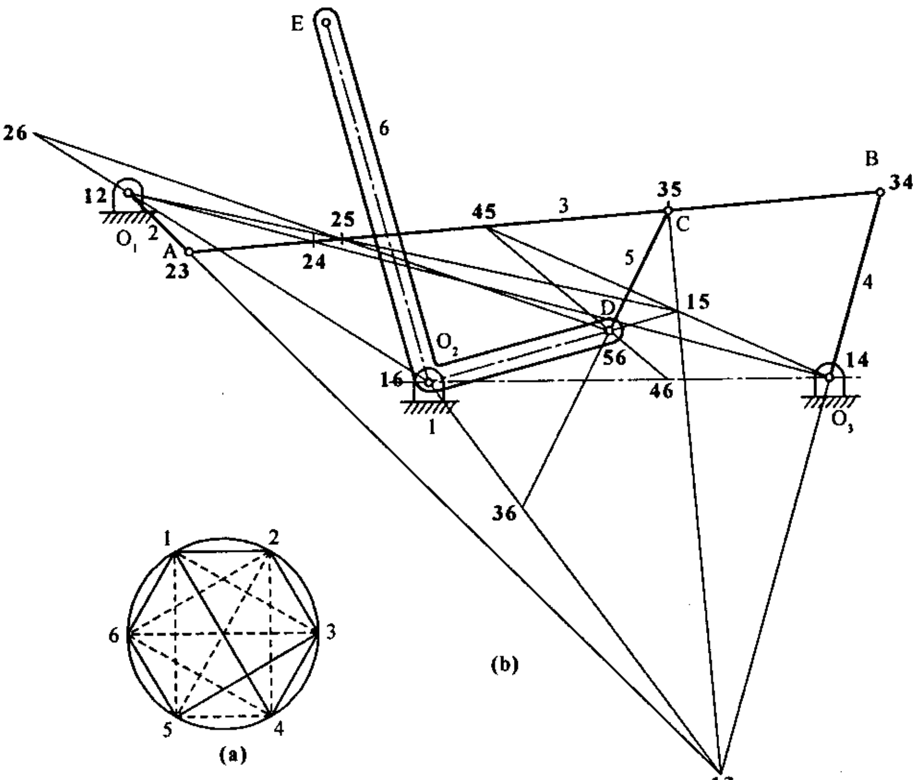

Fig. 4.27

$\therefore$ Velocity of C, $$V_C = V_A \frac{(13-35)}{(13-23)} = 4000 \times \frac{528.61}{696.42} = 3036.2 \text{ mm/s}$$

Consider the links 1, 5 and its centro 15,

$$\frac{V_C}{15-35} = \frac{V_D}{15-56}$$

On measurement, (15-35) = 94.27 mm and (15-56) = 66.63 mm

$\therefore$ Velocity of D, $$V_D = V_C \frac{(15\text{-}56)}{(15\text{-}35)} = 3036.2 \times \frac{66.63}{94.35} = 2144.13 \text{ mm/s}$$

Consider the bell crank lever 6 and its centro 16,

$$\frac{V_D}{16\text{-}D} = \frac{V_E}{16\text{-}E}$$

$\therefore$ Velocity of E, $$V_E = V_D \frac{(16\text{-}E)}{(16\text{-}D)} = V_D \times \frac{O_2E}{O_2D}$$

$$= 2144.13 \times \frac{350}{175} = 4288.3 \text{ mm/s}$$

Example 4.13

Locate all the instantaneous centers of the slider crank mechanism. The length of the crank is 0.3 m, and the length of the connecting rod is 1.5 m. If the crank rotates at 450 rpm clockwise and the crank is inclined at 45° with the inner dead center, find (i) Velocity of the slider, and (ii) Angular velocity of the connecting rod.

Solution :

Number of links $n = 4$

Number of centros $$N = \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$$

Draw the configuration diagram of the given slider crank mechanism with suitable scale. Since there are 4 links, locate the four corners of the centro polygon on the circumference of a circle of arbitrary radius and label the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

Locate the 4 primary centros by inspection. There are three pin joints and one sliding pair. Centros 12, 23 and 34 are the centers of the pin joints and the centro 14 is at infinity in a direction perpendicular to the slider's motion. Draw solid lines in the centro polygon representing the centros whose locations are known (refer fig. 4.28*a*).

Locate the two remaining centros by using kennedy's theorem.

1. From the centro polygon, the side 13 is common between the two triangles 123 and 134. Locate the centro 13 which lies at the intersection of the lines 12-23 and 14-34. Join the centro polygon corners 1, 3 by solid line to represent the centro 13 is located.
2. Similarly, the side 24 is common between the two triangles 124 and 234. Locate the centro 24 which lies at the intersection of the lines 12-14 and 23-34. In this case, to draw a line through centros 12 and 14, it is necessary to move the vertical infinite line

14 to where it will pass through 12. Join the corners 2 and 4 of the centro polygon by solid line. If all the lines of the circle diagram have become solid, it indicates that all centros are determined.

Speed of the crank AB, $n_2 = 450$ rpm

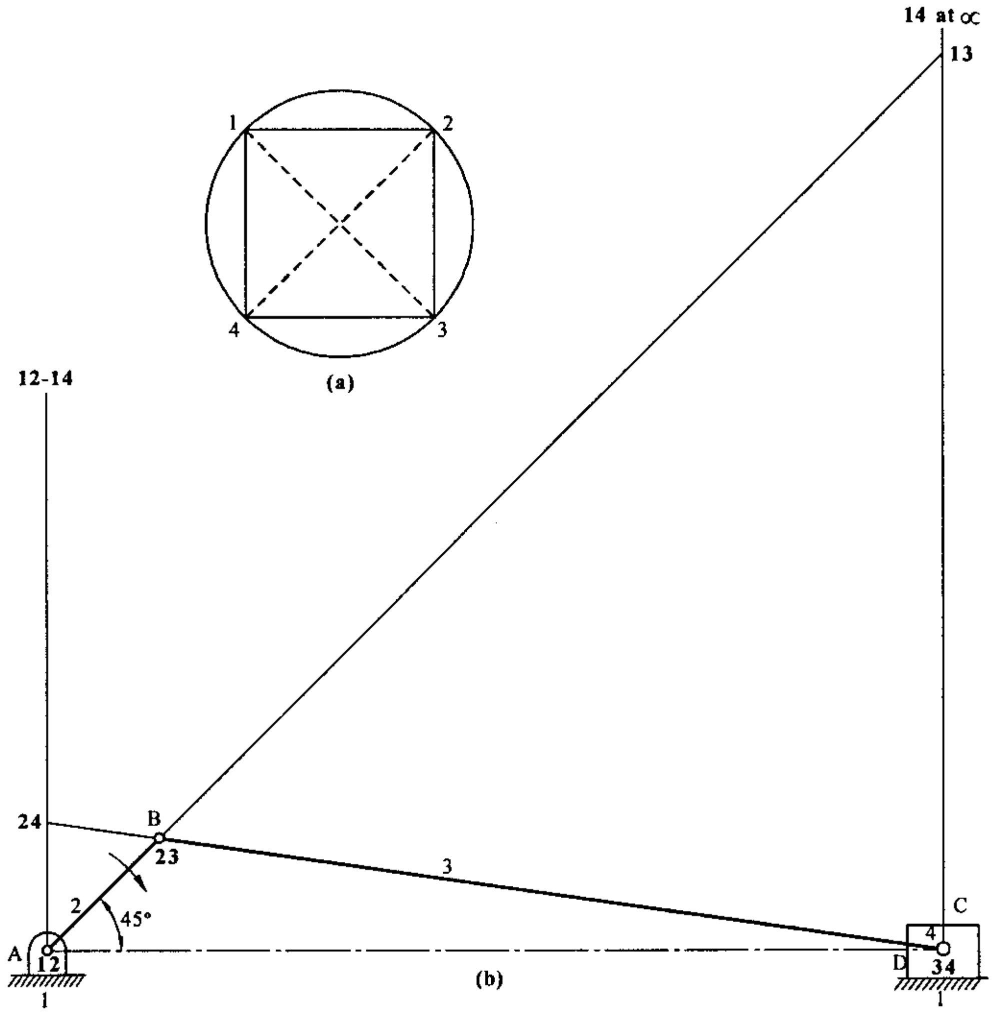

Fig. 4.28

Angular velocity of the crank AB, $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 450}{60} = 47.124$ rad/s

Velocity of B, $V_B = \omega_2 \times AB$

$= 47.124 \times 0.3 = 14.1372$ m/s

Consider the connecting rod BC and its instantaneous center 13

On measurement (refer fig. 4.28b)

$$IB = 13 - 23 = 2.08 \text{ m}$$

$$IC = 13 - 34 = 1.68 \text{ m}$$

We know that, $\frac{V_B}{IB} = \frac{V_C}{IC} = \omega_{BC}$

∴ Velocity of slider C, $V_C = \frac{V_B}{IB} \times IC$

$$= \frac{14.1372}{2.08} \times 1.68 = 11.4185 \text{ m/s}$$

Angular velocity of the connecting rod $\omega_{BC} = \frac{V_B}{IB} = \frac{14.1372}{2.08} = 6.7967 \text{ rad/s}$

Example 4.14

Fig. 4.29 shows a toggle mechanism in which the crank OP rotates at uniform speed of 120 rpm in clockwise direction. Locate all the instant centers of the mechanism and determine the velocity of the slider S. The lengths of various links are: OP = 80 mm, PR = 180 mm, QR = 240 mm and SR = 270 mm.

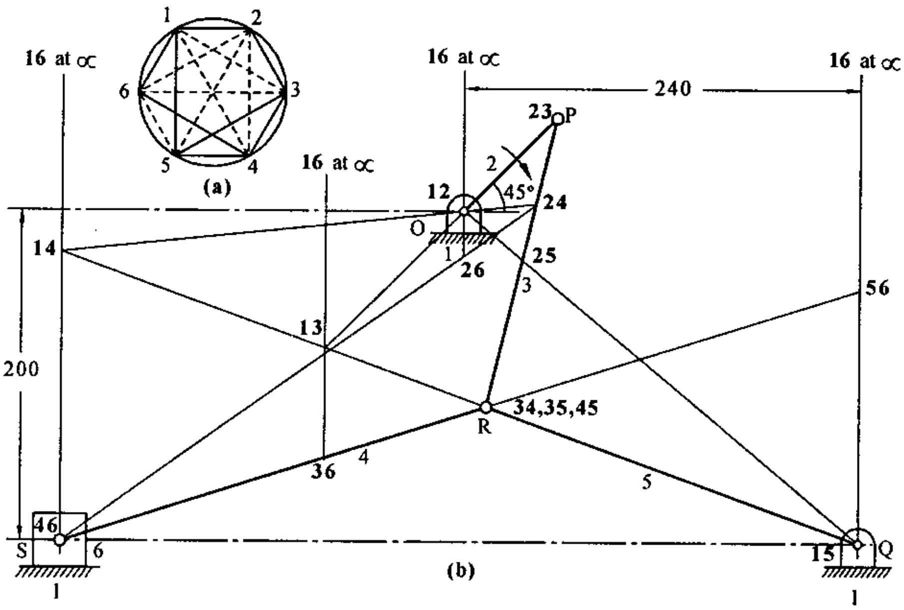

Fig. 4.29

Solution :

Number of links $n = 6$

Number of centros $N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$

Draw the configuration diagram for the given mechanism with suitable scale, since there are 6 links, locate the six corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corner of the polygon.

Locate the 8 primary centers by inspections. There are 7 pin joints and one sliding pair. Centros 12, 23, 34, 45, 35, 46 and 15 are the centers of pin joints and the centro 16 is at infinity in a direction perpendicular to the slider's motion (refer fig. 4.29*b*). Draw solid lines in the centro polygon representing centros whose locations are known (refer fig. 4.29*a*)

Locate the remaining 7 unknown centros by Kennedy's theorem.

Unknown side	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	135	12-23	15-35	13
14	146	134	16-46 (Vertical line through 46)	13-34	14
24	124	234	12-14	23-34	24
25	125	245	12-15	24-45	25
26	126	246	12-16 (Vertical line thorugh 12)	24-46	26
36	136	346	13-16 (Vertical line thorugh 13)	34-46	36
56	156	456	15-16 (Vertical line thorugh 15)	45-46	56

Speed of link 2, $n_2 = 120$ rpm

Angular velocity of link 2, $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times 120}{60} = 4\pi$ rad/s

Velocity of P, $V_p = \omega_2 \times OP = 4\pi \times 80 = 1005.3$ mm/s

Consider the link 3 and its instantaneous center 13,

On measurement (fig. 4.29 *b*),

Distance $IP = 13 - 23 = 197.6$ mm

and $IR = 13 - 34 = 103.6$ mm

We know that, $\frac{V_P}{IP} = \frac{V_R}{IR}$

∴ Velocity of R, $V_R = V_P \times \frac{IR}{IP}$

$= 1005.3 \times \frac{103.6}{197.6} = 527.07$ mm/s

Consider the link 4, and its instantaneous center 14, on measurement (fig. 4.29*b*)

Distance IR = 14 – 34 = 273.6 mm

and IS = 14 – 46 = 174.6 mm

We know that, $\frac{V_R}{IR} = \frac{V_S}{IS}$

∴ Velocity of slider S, $V_S = V_R \times \frac{IS}{IR}$

$= 527.07 \times \frac{174.6}{273.6} = 336.35$ mm/s

Example 2.15

In fig. 4.30, the crank OA is 100 mm long and rotates in a clockwise direction at a speed of 100 rpm. The straight rod BCD rocks on a fixed pivot at C. BC and CD are each 200 mm long and the link AB is 300 mm long. The slider E is driven from D by the rod DE which is 200 mm long. When OA is 30°, below the horizontal as shown, find the velocity of A, B, D and the slider E.

Solution :

Number of links $n = 6$

Number of centros $N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$

Draw the configuration diagram of the given mechanism with suitable scale. Since there are 6 links, locate the six corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corners to every other corner of the polygon.

Locate the 7 primary centros by inspection. There are 6 pin joints and one sliding pair. Centros 12, 23, 34, 45, 56 and 14 are the centers of the pin joints and the centro 16 is at infinity in a direction perpendicular to the slider's motion. Draw solid lines in the centro polygon representing centros whose locations are known (refer fig. 4.30*b*)

Locate the remaining 8 unknown centros by Kennedy's theorem as shown in the table.

Speed of the link 2, $n_2 = 100$ rpm

Angular velocity of the link 2, $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times 100}{60} = 10.472$ rad/s

Velocity of A, $V_A = \omega_2 \times OA$

$= 10.472 \times 0.1 = 1.0472$ m/s

Unknown side	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	134	12-23	14-34	13
15	156	145	16-56 (Vertical line through 56)	14-45	15
24	124	234	12-14	23-34	24
25	125	245	12-15	24-45	25
26	126	256	12-16 (Vertical line through 12)	25-56	26
35	235	345	23-25	34-45	35
36	136	356	13-16 (Vertical line through 13)	35-56	36
46	346	456	34-36	45-56	46

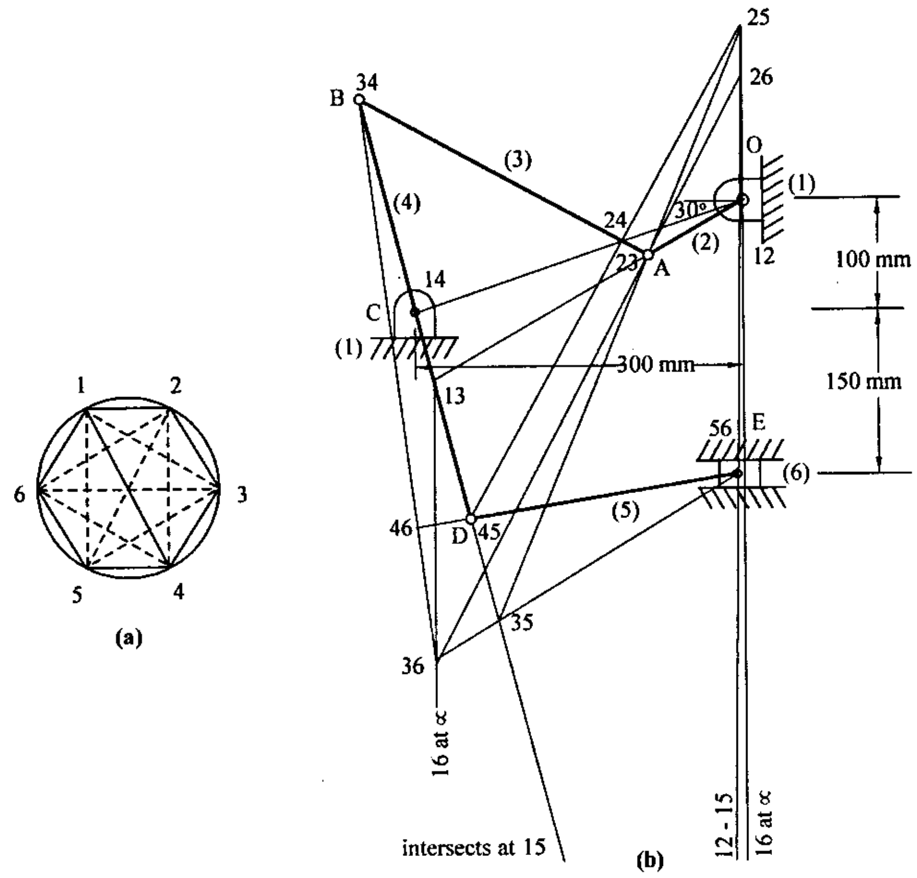

Fig. 4.30

Consider the link 3 and its instantaneous center 13,

On measurement (from fig. 4.30*b*)

Distance $IA = 13 - 23 = 0.23$ m, $IB = 13 - 34 = 0.2675$ m

We know that, $\frac{V_A}{IA} = \frac{V_B}{IB}$

$\therefore$ Velocity of B, $V_B = \frac{V_A}{IA} \times IB = \frac{1.0472}{0.23} \times 0.2675 = 1.218$ m/s

Consider the link 4 and its instantaneous center 14,

On measurement, $IB = 14 - 34 = 0.2$ m, $ID = 14 - 45 = 0.2$ m

We known that, $\frac{V_B}{IB} = \frac{V_D}{ID}$

$\therefore$ Velocity of D, $V_D = \frac{V_B}{IB} \times ID = \frac{1.218}{0.2} \times 0.2 = 1.218$ m/s

Consider the link 5 and its instantaneous center 15,

On measurement, $ID = 15 - 45 = 1.175$ m, $IE = 15 - 56 = 1.2$ m

We known that, $\frac{V_D}{ID} = \frac{V_E}{IE}$

$\therefore$ Velocity of solider E, $V_E = \frac{V_D}{ID} \times IE = \frac{1.218}{1.175} \times 1.2 = 1.244$ m/s

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Example 2.16

In a crank and slotted type quick return motion mechanism shown in fig. 4.31a, the crank rotates at 60 rpm. Determine the velocity of the ram D and the angular velocities of the guiding arm OC and the connecting link CD. Link AB = 200 mm, OC = 600 mm, CD = 500 mm.

Solution :

Number of links $n = 6$

Number of centros $N = \frac{n(n-1)}{2} = \frac{6(6-1)}{2} = 15$

Draw the configuration diagram of the given mechanism with suitable scale. Since there are 6 links, locate the six corners of the centro polygon on the circumference of a circle and lable the corners. Draw thin-dashed lines from each corner to every other corners of the polygon.

Locate the primary centros by inspection. There are 5 pin joints, and two sliding pairs. Centros 12, 14, 23, 45 and 56 are the centers of the pin joints. The centro 16 of the sliding block 6 is at infinity and is perpendicular to the travel of the block 6. The centro 34 of the guide block 3 is at infinity and is perpendicular to the link 4. Draw solid lines in the centro polygon representing centros whose locations are known (refer fig. 4.31b).

Locate the remaining 8 centros using the following table.

Unknown side (centro)	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
15	145	156	14-45	16-56	15
				Move the vertical line 16 to 56	
13	123	134	12-23	14-34	13
				Move the infinite line 34 to 14	
24	234	124	23-34	12-14	24
25	245	125	24-45	12-15	25
26	126	256	12-16	25-56	26
			Move the vertical line 16 to 12		
35	345	235	34-45	23-25	35
			Move the infinite line 34 to 45		
36	356	136	35-56	13-16	36
				Move the vertical line 16 to 13	
46	456	146	45-56	14-16	46
				Move the vertical line 16 to 14	

Speed of the crank $n_2 = 60$ rpm

Angular velocity of the crank $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 60}{60} = 2\pi$ rad/s

Velocity of B, $V_B = \omega_2 \times AB = 2\pi \times 200 = 1256.64$ mm/s

Consider the links 2 and 4 and the common centro 24.

Radio of angular velocity $\dfrac{\omega_4}{\omega_2} = \dfrac{(12-24)}{(14-24)}$

On measurement, distance (12-24) = 185.71 mm and (14-24) = 435.71 mm

$\therefore$ Angular velocity of guiding arm OC, $\omega_4 = \omega_2 \dfrac{(12-24)}{(14-24)}$

$$= 2\pi \times \frac{185.71}{435.71} = 2.678 \text{ rad/s}^2$$

Consider the links 2 and 5 and the common centro 25.

Ratio of angular velocity $\dfrac{\omega_5}{\omega_2} = \dfrac{(12-25)}{(15-25)}$

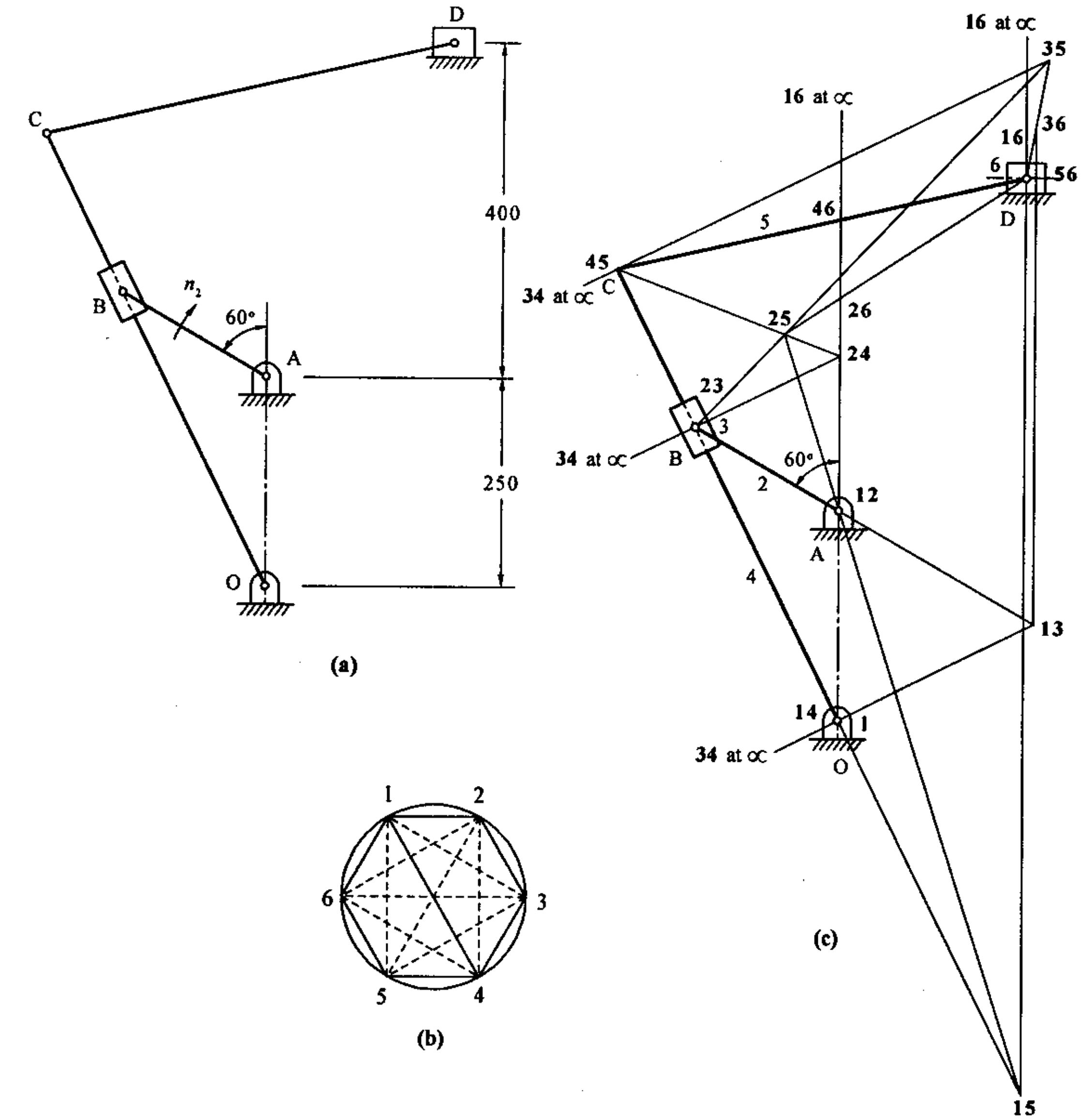

Fig. 4.31

On measurement, distance (12-25) = 221.86 mm and (15-25) = 953.11 mm

$\therefore$ Angular velocity of connecting link CD, $\omega_5 = \omega_2 \dfrac{(12\text{-}25)}{(15\text{-}25)}$

$$= 2\pi \times \frac{221.86}{953.11} = 1.4626 \text{ rad/s}$$

Consider the links 2 and 6 and the common centro 26

$$\frac{V_D}{(12-26)} = \frac{V_B}{(12-23)}$$

On measurement, distance (12-26) = 255.35 and (12-23) = 200 mm

$\therefore$ Velocity of the ram $$V_D = V_B \frac{(12-26)}{(12-23)}$$

$$= 1256.64 \times \frac{255.35}{200} = 1604.4 \text{ mm/s}$$

KLEIN'S CONSTRUCTION

A graphical method for determining the velocity and acceleration of the piston, given by Prof. Klein, is shown in fig. 4.32.

Construction :

1. Draw the slider crank mechanism OCP for the given position of crank OC.
2. Produce the connecting rod PC to meet the vertical through O at M.
3. With PC as diameter draw a circle.
4. With C as center and radius equal to CM draw another circle to cut the previous circle at R and S.
5. Joint SR cutting PC in T and OP in N.

Now OCM is the velocity diagram and OCTN is the acceleration diagram of the mechanism. Then,

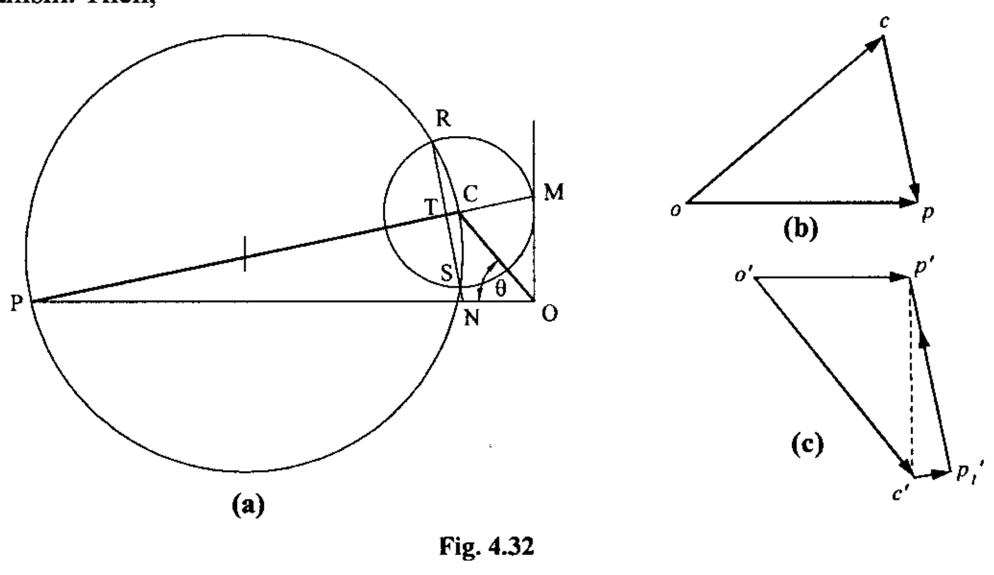

Fig. 4.32

Velocity of the piston $V_p = \omega \times OM$

Acceleration of the piston $A_p = \omega^2 \times ON$

Angular velocity of the connecting rod $\omega_{pc} = \omega \times \frac{CM}{PC}$

Angular accleration of the connecting rod $\alpha_{pc} = \omega^2 \times \frac{TN}{PC}$

Note :

1. To find the velocity of any point D on the connecting rod PC, divide CM at D_1 in the same ratio as D divides CP (refer fig. 4.34)

   i.e., $$\frac{CD_1}{CM} = \frac{CD}{CP}$$

   Velocity of D, $V_D = \omega \times OD_1$

2. To find the acceleration of any point D on the connecting rod PC, draw a line from the point D parallel to PO which intersects CN at D_2 (refer fig. 4.34)

   Acceleration of D, $A_D = \omega^2 \times OD_2$

Proof :

The velocity and acceleration diagram for the slider crank mechanism is shown in fig. 4.32 (b) and (c) respectively. In order to prove the quadrilateral OCTN represents acceleration diagram for this mechanism, it is necessary to prove that the quadrilateral OCTN is similar to the quadrilateral $o'c'p_1'p'$ i.e., (1) The sides of quadrilateral OCTN and $o'c'p_1'p'$ should be parallel, (2) Ratio of adjacent sides of quadrilateral should be equal to the ratio of corresponding adjacent sides of another quadrilateral.

As SR is perpendicular to CP, it will be perpendicular to CT also. Consider two quadrilateral OCTN and $o'c'p_1'p'$.

ON is parallel to $o'p'$ by construction

OC is parallel to $o'c'$ by construction

CT is parallel to $c'p_1'$ by construction

TN is parallel to $p_1'p'$ by construction

Hence, first condition is fulfilled.

Join PR and CR. From triangles CPR and CTR,

angle CRP = angle RTC = right angles

angle RCP = angle RCT = common angle

$\therefore$ angle CPR = angle CRT

Therefore two triangles CPR and CTR are similar.

$$\therefore \quad \frac{CR}{CP} = \frac{CT}{CR}$$

$$CT = \frac{CR^2}{CP} = \frac{CM^2}{CP} \qquad (\because \ CR = CM)$$

Dividing both sides by OC,

$$\frac{CT}{OC} = \frac{CM^2}{CP \times OC} \qquad \text{..... (1)}$$

Now, $$\frac{c'p_l'}{o'c'} = \frac{\text{Centripedal acceleration of CP}}{\text{Acceleration of crank C}}$$

$$= \frac{V_{cp}^2 / CP}{\omega^2 \times OC} = \frac{\omega^2 \times CM^2 / CP}{\omega^2 \times OC}$$

$$\frac{c'p_l'}{o'c'} = \frac{CM^2}{CP \times OC} \qquad \text{..... (2)}$$

From equation (1) and (2)

$$\frac{CT}{OC} = \frac{c'p_l'}{o'c'}$$

Hence the second condition is fulfilled. Therefore quadrilateral OCTN is similar to quadrilateral $o'c'p'_l\,p'$. Thus OCTN represents the acceleration diagram to some scale.

Example 4.17

Determine the velocity and acceleration of the piston by Klein's construction for a steam engine to the following specifications :

Stroke of piston = 600 mm

Ratio of length of the connecting rod to crank length = 5

Speed of the engine = 450 rpm clockwise

Position of the crank = 45° with inner dead center

Solution :

Radius of the crank $r = \frac{\text{Stroke}}{2} = \frac{600}{2} = 300 \text{ mm} = 0.3 \text{ m}$

Length of the connecting rod $l = 5 \times 0.3 = 1.5 \text{ m}$

Angular velocity $\omega = \dfrac{2\pi n}{60} = \dfrac{2\pi \times 450}{60} = 47.124$ rad/s

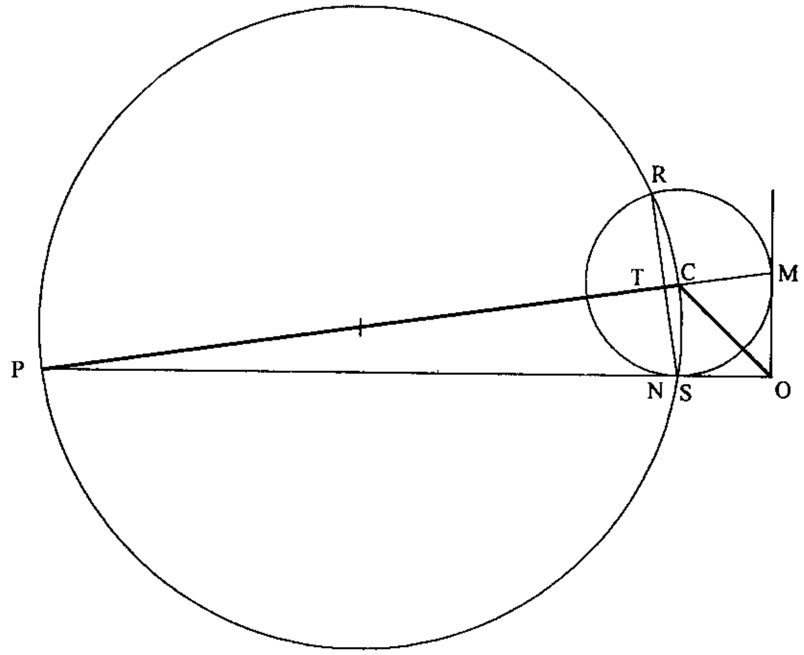

Fig. 4.33

Construction : *(Refer fig. 4.33)*

1. Draw the slider crank mechanism OCP for the given position of the crank with some suitable scale.
2. Produce the connecting rod PC to meet the vertical at O in M.
3. With PC as diameter draw a circle.
4. With C as center and radius equal to CM draw another circle to cut the previous circle at R and S.
5. Join SR cutting PC in T and OP in N. In this example N and S are at the same point.

On measuring, OM = 0.2475 m and ON = 0.2025 m

Velocity of the piston $V_p = \omega \times OM = 47.124 \times 0.2475$

$= 11.66$ m/s

Acceleration of the piston $= \omega^2 \times ON = 47.124^2 \times 0.2025$

$= 449.69$ m/s^2

Example 4.18

The stroke of the reciprocating steam engine is 400 mm and the length of the connecting rod is 600 mm. The crank rotates at 300 rpm in the clockwise direction and the crank is inclined at 30° with the inner dead center. The center of gravity of the connecting rod is 240 mm away from the crank end. By Klein's construction determine the following:

i) Velocity and acceleration of the piston.

ii) Angular velocity and angular acceleration of the connecting rod.

iii) Velocity and acceleration of the center of gravity of the connecting rod.

Solution :

Stroke $S = 400$ mm $= 0.4$ m, $l = 600$ mm $= 0.6$ m, $\theta = 30°$, $n = 300$ rpm

Radius of the crank $$r = \frac{S}{2} = \frac{0.4}{2} = 0.2 \text{ m}$$

Angular velocity $$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 300}{60} = 31.4 \text{ rad/s}$$

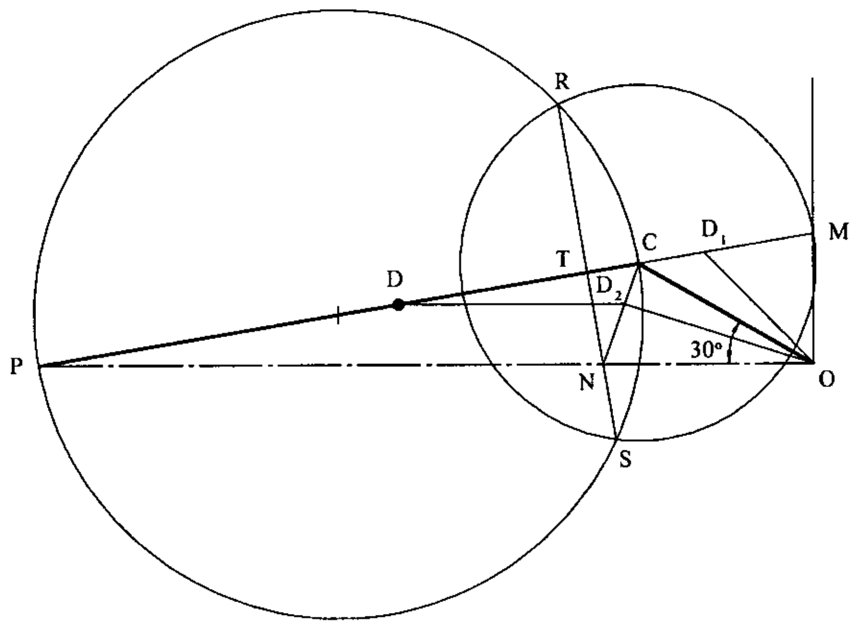

Fig. 4.34

Construction : *(Refer fig. 4.34)*

1. Draw the slider crank mechanism OCP for the given position of the crank with suitable scale.

2. Produce the connecting rod PC to meet the vertical at O in M.
3. With PC as diameter draw a circle.
4. With C as center and radius equal to CM draw another circle to cut the previous circle at R and S.
5. Join SR cutting PC in T and OP in N.
6. Locate the point D at a distance of 0.24 m from C on the connecting rod PC and draw a line from D parallel to PO which intersects the line joining CN at D_2.
7. Locate the point D_1, on the line CM such that

$$\frac{CD_1}{CM} = \frac{CD}{CP}$$

On measurement,

OM = 0.13 m, ON = 0.21 m, CM = 0.175 m, TN = – 0.095 m,

OD_1 = 0.155 m, OD_2 = 0.195 m

Velocity of the piston $V_p = \omega \times OM$

$= 31.4 \times 0.13 = 4.082$ m/s

Acceleration of the piston $A_p = \omega^2 \times ON$

$= 31.4^2 \times 0.21 = 207.05$ m/s²

Angular velocity of the connecting rod $\omega_{pc} = \omega \times \frac{CM}{PC}$

$= 31.4 \times \frac{0.175}{0.6} = 9.16$ rad/s

Angular acceleration of the connecting rod $\alpha_{pc} = \omega^2 \times \frac{TN}{PC}$

$= 31.4^2 \times \frac{(-0.095)}{0.6} = -156$ rad/s²

Velocity of the point D, $V_D = \omega \times OD_1$

$= 31.4 \times 0.155 = 4.867$ m/s

Acceleration of the point D, $A_D = \omega^2 \times OD_2$

$= 31.4^2 \times 0.195 = 192.26$ m/s²

Example 4.19

The length of the crank and connecting rod of a reciprocating engine are 200 mm and 800 mm respectively. The crank is rotating at a uniform speed of 480 rpm. Using Klein's construction find: (i) Acceleration of the piston, (ii) Acceleration of the middle point of the connecting rod and (iii) Angular acceleration of the connecting rod when the crank has turned through 45° from the inner dead center

Data : $r = 200$ mm = 0.2 m, $l = 800$ mm = 0.8 m, $n = 480$ rpm, $\theta = 45°$

Solution :

Angular velocity of the crank $\omega = \frac{2\pi n}{60} = \frac{2\pi \times 480}{60} = 50.265$ rad/s

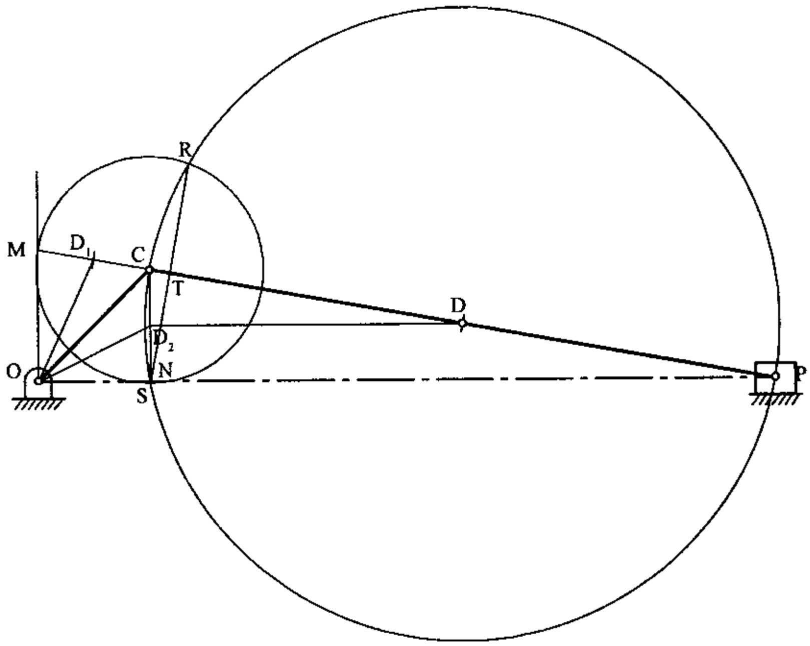

Fig. 4.35

Construction : *(Refer fig. 4.35)*

1. Draw the slider crank mechanism OCP for the given position of the crank with suitable scale.
2. Produce the connecting rod PC to meet the vertical at O in M.
3. With PC as diameter, draw a circle.
4. With C as center and CM as radius, draw another circle to cut the previous circle at R and S.
5. Join SR cutting PC at T and OP at N.
6. Locate the center point D of the connecting rod and draw a line from D parallel to PO which intersects the line joining CN at D_2.
7. Locate the point D_1 on the line CM such that

$$\frac{CD_1}{CM} = \frac{CD}{CP} = \frac{1}{2}$$

On measurement from fig. 4.35

Distance OM = 0.167 m, ON = 0.142 m, CM = 0.144 m, TN = – 0.139 m, OD_1 = 0.1696 m, OD_2 = 0.158 m.

Velocity of the piston $V_P = \omega \times \text{OM}$

$= 50.265 \times 0.167 = 8.394$ m/s

Acceleration of the piston $A_P = \omega^2 \times \text{ON}$

$= 50.265^2 \times 0.142 = 358.78$ m/s^2

Velocity of the middle point D $V_D = \omega \times \text{OD}_1$

$= 50.265 \times 0.1696 = 8.525$ m/s

Acceleration of the middle point D $A_D = \omega^2 \times \text{OD}_2$

$= 50.265^2 \times 0.158 = 3.99.2$ m/s^s

Angular velocity of the connecting rod $\omega_{PC} = \omega \times \dfrac{\text{CM}}{\text{PC}}$

$$= 50.265 \times \frac{0.144}{0.8} = 9.05 \text{ rad/s}$$

Angular acceleration of the connecting rod $a_{PC} = \omega^2 \times \dfrac{\text{TN}}{\text{PC}}$

$$= 50.265^2 \times \frac{(-0.139)}{0.8} = -438.99 \text{ rad/s}^2$$

Example 4.20

In a slider crank mechanism, the length of crank and connecting rod are 125 mm and 500 mm respectively. The center of gravity G of the connecting rod is 275 mm from the slider. The crank speed is 600 rpm clockwise. The crank makes 45° from inner dead center. Locate all the instantaneous centers and find velocity of slider, velocity of point G and angular velocity of connecting rod. By Klein's construction, determine the acceleration of the slider and the point G.

(VTU, Jan 2006)

Data :

$r = 125$ mm $= 0.125$ m, $l = 500$ mm $= 0.5$ m, $n_2 = 600$ rpm, $\theta = 45°$,

PG $= 275$ mm $= 0.275$ m

Solution :

Number of links $n = 4$

Number of centros $N = \dfrac{n(n-1)}{2} = \dfrac{4(4-1)}{2} = 6$

Draw the configuration of the given slider crank mechanism with suitable scale. Since there are 4 links, locate the four corners of the centro polygon on the circumference of a circle and label the corners. Draw thin dashed lines from each corner to every other corners of the polygon.

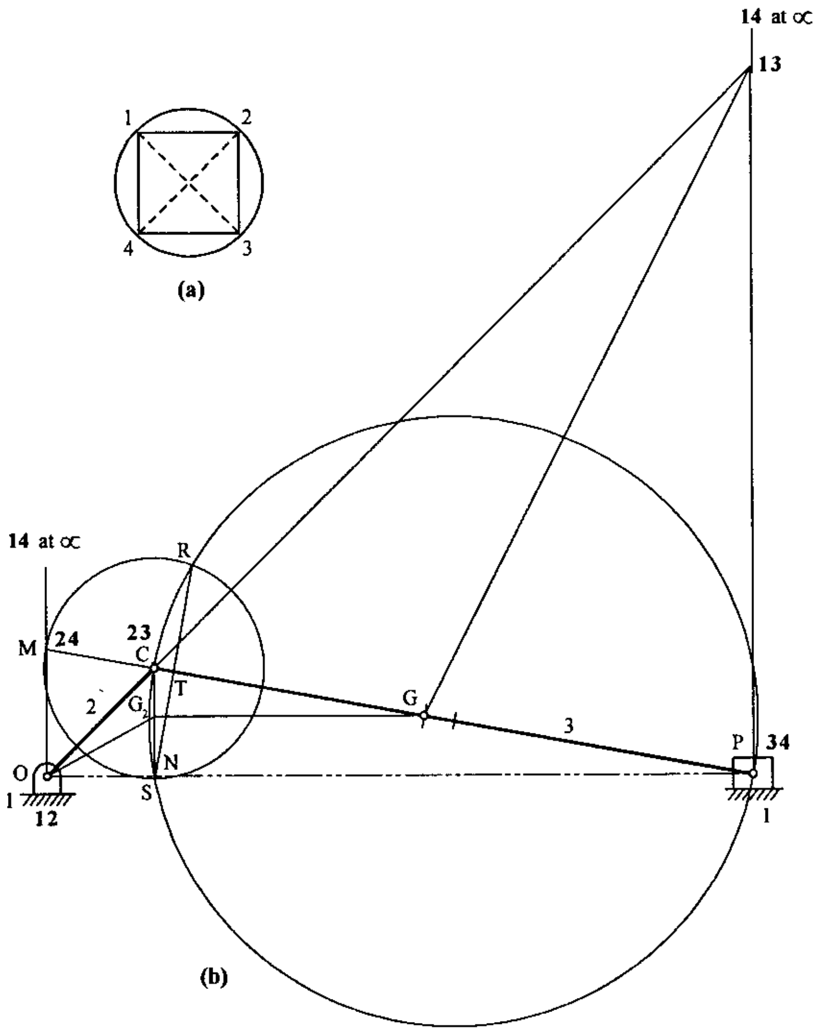

Fig. 4.36

Locate the 4 primary centros by inspection. There are 3 pin joints and one sliding pair. Centros 12, 23 and 34 are the centers of the pin joints. The centro 14 is at infinity in a direction perpendicular to the slider's motion. Draw solid lines in the centro polygon representing the centros whose locations are known (fig. 4.36 a).

Locate the two remaining centros by using Kennedy's theorm as shown in the table.

Speed of the crank OC, $n_2 = 600$ rpm

Angular velocity of the crank OC, $\omega_2 = \dfrac{2\pi n_2}{60} = \dfrac{2\pi \times 600}{60} = 62.832$ rad/s

Velocity of C, $V_C = \omega_2 \times OC = 62.832 \times 0.125 = 7.854$ m/s

Unknown side (centro)	Triangle 1	Triangle 2	Extension line 1	Extension line 2	Line to be made solid in the centro polygon
13	123	134	12-23	14-34	13
24	124	234	12-14	23-34	24
			(Move the vertical line 14 to 12)		

Consider the link 3 and its centro 13

$$\frac{V_C}{13-23} = \frac{V_P}{13-34} = \frac{V_G}{13-G} = \omega_{PC}$$

On measurement, distance (13-23) = 0.696 m, (13-34) = 0.581 m and (13-G) = 0.597 m

$\therefore$ Velocity of the slider $V_P = V_C \dfrac{(13-34)}{(13-23)} = 7.854 \times \dfrac{0.581}{0.696} = 6.55$ m/s

Also, velocity of point G, $V_G = V_C \dfrac{(13-G)}{(13-23)} = 7.854 \times \dfrac{0.597}{0.696} = 6.7349$ m/s

Angular velocity of the connecting rod $\omega_{PC} = \dfrac{V_C}{13-23} = \dfrac{7.854}{0.696} = 11.284$ m/s

Draw the Klein's construction as shown in fig. 4.36

On measurement, distance ON = 89.04 mm and OG_2 = 101.16 mm

Acceleration of the piston $A_P = \omega^2 \times ON = 62.832^2 \times 0.089 = 351.36$ m/s^2

Acceleration of the point G, $A_G = OG_2 = 62.832^2 \times 0.1012 = 399.52$ m/s^2

Determination of Velocity and Acceleration of the Piston by Analytical Method

Fig. 4.37 shows the configuration of the slider crank mechanism.

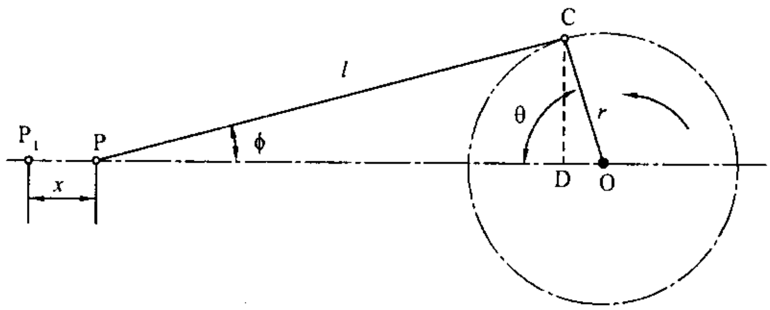

Fig. 4.37

Let ω = Angular velocity of the crank OC

ω_c = Angular velocity of the connecting rod PC

V_p = Velocity of the piston P

A_p = Acceleration of the piston

α_c = Angular acceleration of the connecting rod

l = Length of the connecting rod

r = Length of the crank

n_1 = Ratio of the length of the connecting rod to the crank length, i.e., $n_l = l/r$

θ = Inclination of the crank and from the inner dead center

ϕ = Inclination of the connecting rod with the line of stroke

x = Displacement of the piston from the end position

Draw CD perpendicular to the line of stroke OP

from the figure, $x = PP_1 = OP_1 - OP$

but $OP_1 = OC + CP = r + l$

and $OP = OD + DP = r\cos\theta + l\cos\phi$

$$\therefore \quad x = r + l - r\cos\theta - l\cos\phi$$

$$= r(1-\cos\theta) + l(1-\cos\phi) \qquad \text{..... (1)}$$

Now CD is the common side of two triangles OCD and DPC,

therefore, $CD = r\sin\theta = l\sin\phi$

or $$\sin\phi = \frac{r\sin\theta}{l} = \frac{\sin\theta}{n_1} \qquad \text{..... (2)}$$

but $$\cos\phi = \sqrt{1-\sin^2\phi} = \sqrt{1-\frac{\sin^2\theta}{n_1^2}} = \frac{1}{n_1}\sqrt{n_1^2-\sin^2\theta}$$

Substituting the value of $\cos\phi$ in equation (1) we have

$$x = r(1-\cos\theta) + l\left\{1-\frac{\sqrt{n_1^2-\sin^2\theta}}{n_1}\right\}$$

$$= r(1-\cos\theta) + l - \frac{l}{n_l}\sqrt{n_1^2-\sin^2\theta}$$

$$= r(1-\cos\theta) + n_l r - r\sqrt{n_1^2-\sin^2\theta} \qquad \left(\text{since } \frac{l}{r} = n_1\right)$$

$$= r\left(1-\cos\theta + n_l - \sqrt{n_l^2-\sin^2\theta}\right)$$

Differentiating the above equation with respect to time,

$$V_p = \frac{dx}{dt} = \frac{d\theta}{dt} \cdot \frac{dx}{d\theta} = \omega \frac{dx}{d\theta} \qquad \left(\text{since } \frac{d\theta}{dt} = \omega\right)$$

$$= \omega \times \frac{d}{d\theta} \left\{ r\left(1 - \cos\theta + n_l - \sqrt{n_l^2 - \sin^2\theta}\right) \right\}$$

$$V_p = \omega r \left[\sin\theta + \frac{\sin 2\theta}{2\sqrt{n_1^2 - \sin^2\theta}} \right] \qquad \text{..... (3)}$$

Differentiating the above equation with respect to time,

$$A_p = \frac{dV_p}{dt} = \frac{d\theta}{dt} \cdot \frac{dV_p}{d\theta}$$

$$= \omega \frac{d}{d\theta} \left[\omega r \left\{ \sin\theta + \frac{\sin 2\theta}{2\sqrt{n_1^2 - \sin^2\theta}} \right\} \right]$$

$$A_p = \omega^2 r \left[\cos\theta + \frac{n_1^2 \cos 2\theta + \sin^4\theta}{2\sqrt{n_1^2 - \sin^2\theta}} \right] \qquad \text{..... (4)}$$

Since $\sin^2\theta$ and $\sin^4\theta$ are very small compared to n_l^2 thus, the equation (3) and (4) becomes,

$$V_p \approx \omega r \left[\sin\theta + \frac{\sin 2\theta}{2n_1} \right] \qquad \text{..... (5)}$$

$$A_p \approx \omega^2 r \left[\cos\theta + \frac{\cos 2\theta}{n_1} \right] \qquad \text{..... (6)}$$

The angular velocity and angular acceleration of the connecting rod are calculate as follows:

Consider the equation (2), $\sin\phi = \dfrac{\sin\theta}{n_1}$

differentiating both sides with respect to time, we get

$$\cos\phi \cdot \frac{d\phi}{dt} = \frac{\cos\theta}{n_1} \cdot \frac{d\theta}{dt}$$

$$\cos\phi \cdot \omega_c = \frac{\cos\theta}{n_1} \cdot \omega \qquad \left(\text{since } \frac{d\phi}{dt} = \omega_c\right)$$

$$\therefore \quad \omega_c = \frac{\omega}{n_1} \times \frac{\cos\theta}{\cos\phi}$$

but, $\qquad \cos\phi = \dfrac{1}{n_1}\sqrt{n_1^2 - \sin^2\theta}$

$$\therefore \quad \omega_c = \frac{\omega}{n_1} \times \frac{n_1 \cos\theta}{\sqrt{n_1^2 - \sin^2\theta}} = \frac{\omega\cos\theta}{\sqrt{n_1^2 - \sin^2\theta}}$$

neglecting the value of $\sin^2\theta$, we have

$$\omega_c \approx \frac{\omega\cos\theta}{n_1} \qquad \text{..... (7)}$$

and

$$\alpha_c = \frac{d\omega_c}{dt} = \frac{d\theta}{dt} \cdot \frac{d\omega_c}{d\theta}$$

$$= -\frac{\omega^2 \sin\theta\left(n_1^2 - 1\right)}{\left(n_1^2 - \sin^2\theta\right)^{3/2}}$$

neglecting the value of 1 and $\sin^2\theta$ being very small compared to n_l^2, we get

$$\alpha_c \approx -\frac{\omega^2}{n_1}\sin\theta \qquad \text{..... (8)}$$

Example 4.21

The crank of a reciprocating engine is 90 mm long, the connecting rod is 360 mm long and the crank rotates at 150 rpm clockwise. Find the velocity and acceleration of the piston and the angular velocity and angular acceleration of the connecting rod when the angle which the crank makes with the inner dead centre is 30°.

Solution :

$r = 90$ mm $= 0.09$ m, $l = 360$ mm $= 0.36$ m, $n_1 = \frac{l}{r} = \frac{0.36}{0.9} = 4$, $\theta = 30°$, $n = 150$ rpm

Angular velocity of crank $\omega = \frac{2\pi n}{60} = \frac{2\pi \times 150}{60} = 15.7$ rad/s

Velocity of piston $V_p = \omega r\left[\sin\theta + \frac{\sin 2\theta}{2n_l}\right] = 15.7 \times 0.09\left[\sin 30 + \frac{\sin 60}{2\times 4}\right]$

$= 0.8595$ m/s

Acceleration of piston $A_p = \omega^2 r\left[\cos\theta + \frac{\cos 2\theta}{n_1}\right] = 15.7^2 \times 0.09\left[\cos 30 + \frac{\cos 60}{4}\right]$

$= 21.985$ m/s^2

Angular velocity of connecting rod $\omega_c = \frac{\omega}{n_1}\cos\theta = \frac{15.7}{4} \times \cos 30 = 3.399$ rad/s

Angular acceleration of connecting rod $\alpha_c = -\frac{\omega^2}{n_1}\sin\theta = -\frac{15.7^2}{4}\sin 30$

$= -30.81$ rad/s^2

REVIEW QUESTIONS

1. Define the term, instantaneous center of rotation.
2. State and explain Kennedy's theorm.
3. State and prove Kennedy's three centers in line theorm. **(VTU, Jan 2006)**
4. Locate the instantaneous center of the following joints.
   i) Revolute joint, ii) Prismatic joint, iii) Cam-pair contact.
5. List the steps in Klein's construction to determine the velocity and the acceleration of the piston in a slider-crank mechanism.
6. What is instantaneous center? Locate all the instantaneous center of a single slider crank mechanism and show how the velocity of the slider is determined. **(VTU, Jan 2005)**
7. Explain with neat diagram how the acceleration of the slider and the connecting rod of a slider crank mechanism can be determined using Klein's construction. **(VTU, July 2005)**

EXERCISE - 4

1. Locate all the instant centers for the mechanism shown in fig. 4.38 and fig. 4.39.

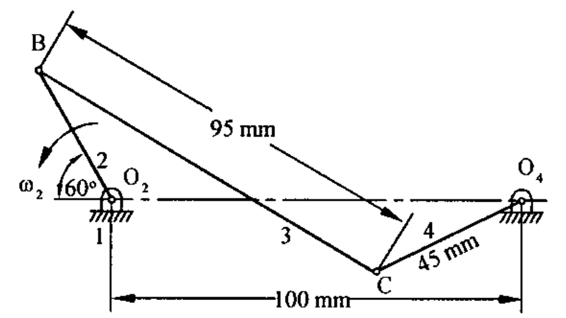

Fig. 4.38

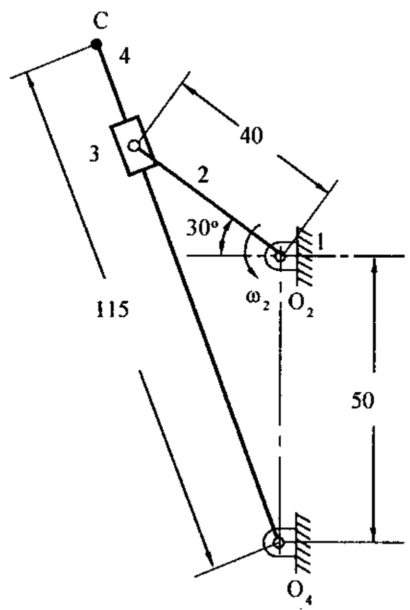

Fig. 4.39

2. Locate all the instant centers of the linkage shown in fig. 4.40.
   AB = 35 mm, BD = 100 mm, DE = 20 mm, BC = 20 mm, CF = 50 mm.

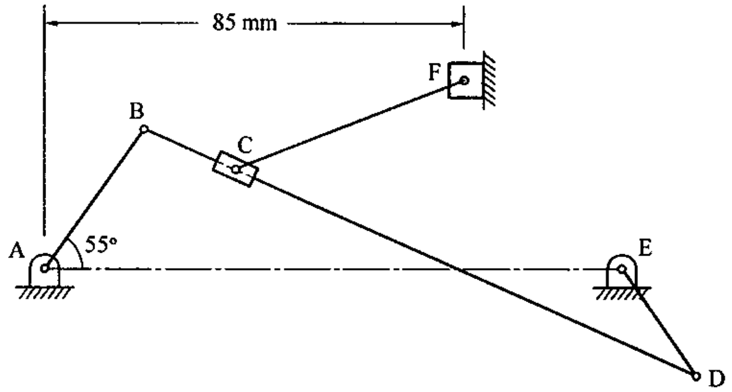

Fig. 4.40

3. For the four bar mechanism shown in fig. 4.41 locate all the instant centers. Determine the angular velocity of the link CD if the crank AB rotates at 3 rad/s. Length AD = 135 mm, AB = 75 mm, BC = 115 mm and CD = 135 mm. [**Ans.** 2.6 rad/s]

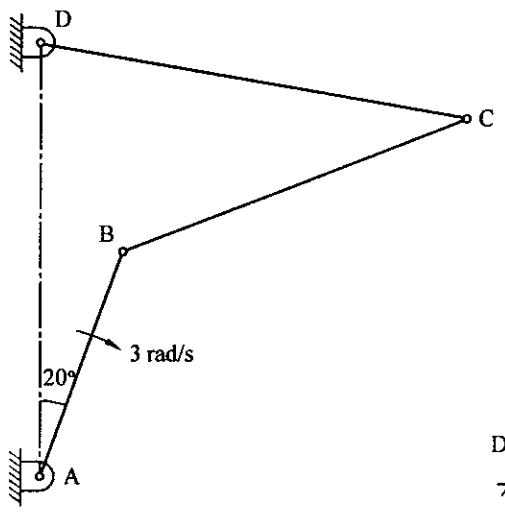

Fig. 4.41

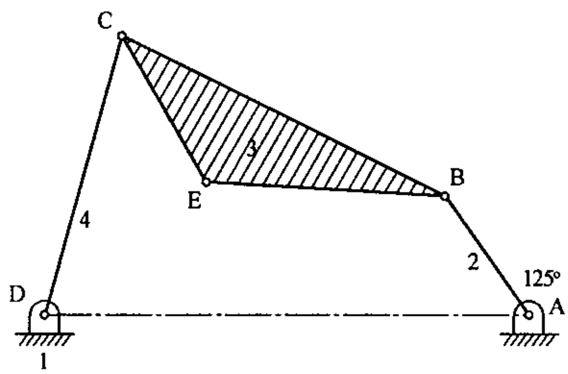

Fig. 4.42

4. Locate all the instant centers in the mechanism shown in fig. 4.42. If the link 2 is turning clockwise at the rate of 60 rad/s, determine the linear velocity of points C and E using instant centers. Lengths Ad = 100 mm, AB = 30 mm, BC = 75 mm, CD = 60 mm, CE = 35 mm and EB = 50 mm.

5. Solve the probelms 6, 7 and 8 by instantaneous center method.

6. Determine the velocity and acceleration of the piston in a slider crank mechanism by Klein's construction to the following specifications. Stroke = 300 mm, Ratio of length of connecting rod to crank = 4, Speed of the engine = 300 rpm, Position of crank = 45° with inner dead center. **(VTU, Jan. 2007)**

   [**Ans.** 3.9212 m/s, 104. 683 m/s^2]

7. The crank of a reciprocating engine is 60 mm long and connecting rod is 240 mm long. The crank rotates at 400 rpm. Find the velocity and acceleration of the piston and the angular velocity and angular acceleration of the connecting rod when the crank is 30° from inner dead center by Klein's construction **(VTU, Jan. 2005)**

   [**Ans.** 1.529 m/s, 104.33 m/s^2, 9.069 rad/s, – 219.32 rad/s^2]

8. In a reciprocating engine, length of crank is 250 mm and length of connecting rod is 1000 mm. The crank rotates at a uniform speed of 300 rpm clockwise. By Klein's construction determine (i) Velocity of piston and angular velocity of connecting rod. (ii) Acceleration of piston and angular acceleration of connecting rod. The crank is at 30° from inner dead center. **(VTU, July 2004)**

   [**Ans.** 4.777 m/s, 6.802 rad/s, 7.7835 m/s^2, 123.37 rad/s^2]

9. The mechanism of a wrapping machine, is as shown in Fig. 4.43 has the following dimensions: O_1A = 100 mm, AC = 700 mm, BC = 200 mm, O_3C = 200 mm, O_2E = 400 mm, O_2D = 200 mm and BD = 150 mm. The crank O_1A rotates at uniform angular velocity of 100 rad/sec. Find the velocity of the point E of the bell crank lever by instantaneous centre method. **(VTU, Jan 2008)**

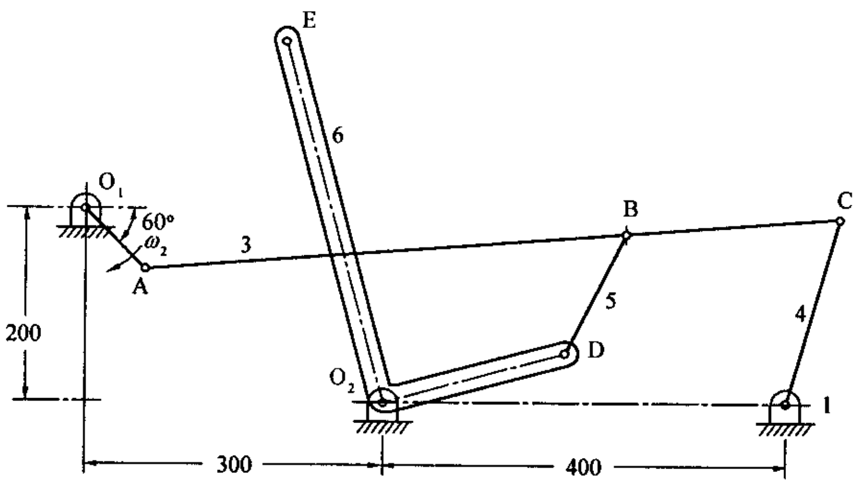

Fig. 4.43

5

VELOCITY AND ACCELERATION ANALYSIS - ANALYTICAL APPROACH

The analytical solution techniques are easier and can be conveniently programmed on a digital computer for repetitive or extensive analysis. The geomentrical constraints associated with mechanisms can be formulated using vector displacement, velocity and acceleration closure equations.

Closure equation

The closure condition simply expresses the condition that a loop of a linkage closes on itself. It is convenient to represent the terms in the closure equations by vectors. For the four-bar linkage mechanism shown in fig. 5.1, the closure equations would be

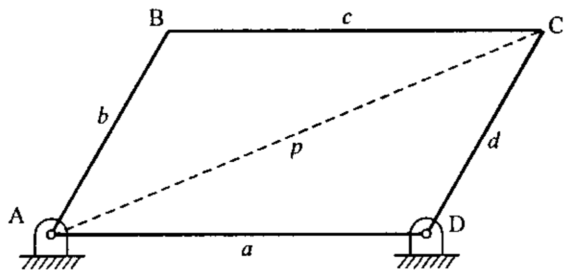

Fig. 5.1

$$\bar{p} = \bar{a} + \bar{d} = \bar{b} + \bar{c} \quad \text{or} \quad \bar{a} = \bar{b} + \bar{c} + \bar{d}$$

where $\bar{a}, \bar{b}, \bar{c}$ and $\bar{d}$ be the position vectors of links DA, AB, BC, and CD respectively.

Raven's approach

The two dimensional kinematics problem can be easily solved by using complex algebra. The complex algebra formulation provides the advantages of increased accuracy and suitable for digital computer computation. The complex algebra approach to velocity and acceleration analysis leads to a set of linear equations, which can be solved in quite straight forward. The method illustrated in the following examples were developed by Raven. In Raven's method, the links of a mechanism are replaced by vectors, and the vector equations are obtained by equating the resultant of the vectors around every closed loop to zero.

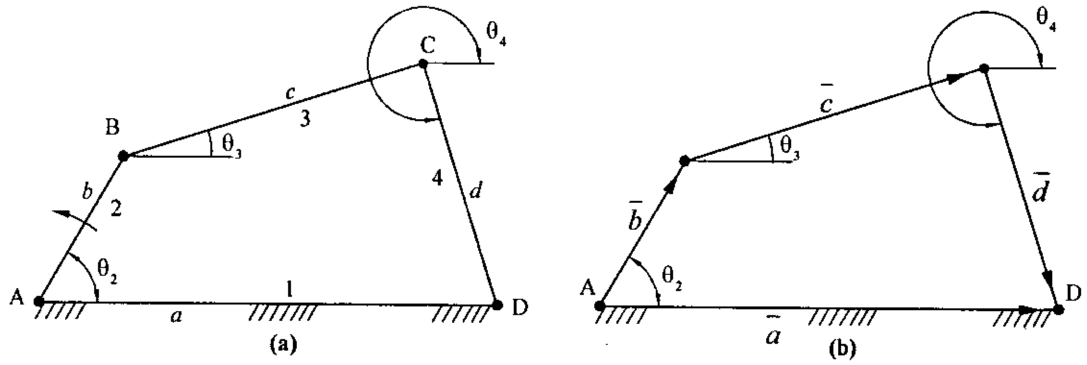

Fig. 5.2

Four-bar mechanism

To illustrate Raven's approach, let us analyse a four bar mechanism shown in fig. 5.2*a*. Let a, b, c, and d denote the length of links, and θ_1, θ_2, θ_3 and θ_4 denote the angular position of links 1, 2, 3, and 4 respectively. The angles are considered as positive when measured counterclockwise as shown. Let $\bar{a}$, $\bar{b}$, $\bar{c}$ and $\bar{d}$ be the position vectors of the links a, b, c, and d respectively (refer fig. 5.2*b*). The loop – closure equations is

$$\bar{b} + \bar{c} + \bar{d} = \bar{a} \qquad \text{.....(1)}$$

where $\bar{a}$ has constant magnitude and direction (fixed link). The vector $\bar{b}$ has constant magnitude and its direction θ_2 varies, which is the input angle. If link 2 is the driving link, then θ_2, ω_2 and α_2 are known quantities. The vector $\bar{c}$ and $\bar{d}$ has unknown magnitude and direction. Expressing the vectors in complex form,

$$be^{i\theta 2} + ce^{i\theta 3} + de^{i\theta 4} = a\,e^{i\theta l} = a \qquad (\because\ \theta_1 = 0) \qquad \text{.....(2)}$$

Differentiating the equation (2) with respect to time, we get

$$ib\frac{d\theta_2}{dt}e^{i\theta 2} + ic\frac{d\theta_3}{dt}e^{i\theta 3} + id\frac{d\theta_4}{dt}e^{i\theta 4} = 0 \qquad (\because\ a \text{ is a fixed link}) \qquad \text{.....(3)}$$

Let $\dfrac{d\theta_2}{dt} = \omega_2$, $\dfrac{d\theta_3}{dt} = \omega_3$, and $\dfrac{d\theta_4}{dt} = \omega_4$,

∴ Equation (3) becomes

$$ib\,\omega_2\,e^{i\theta 2} + ic\,\omega_3\,e^{i\theta 3} + id\,\omega_4\,e^{i\theta 4} = 0 \qquad \text{.....(4)}$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (4), we get

$$ib\,\omega_2(\cos\theta_2 + i\sin\theta_2) + ic\,\omega_3(\cos\theta_3 + i\sin\theta_3) + id\,\omega_4(\cos\theta_4 + i\sin\theta_4) = 0 \qquad \text{.....(5)}$$

The real part of the equation (5) is,

$$R_v = -b\,\omega_2\sin\theta_2 - c\omega_3\sin\theta_3 - d\omega_4\sin\theta_4 = 0 \qquad \text{.....(6)}$$

The imaginary part of the equation (5) is,

$I_v = b\,\omega_2 \cos\theta_2 + c\omega_3 \cos\theta_3 + d\omega_4 \cos\theta_4 = 0$ ..... (7)

Solving the simultaneous equations (6) and (7), we get

Angular velocity of link 3, $\omega_3 = -\dfrac{b\,\sin(\theta_2-\theta_4)}{c\,\sin\,(\theta_3-\theta_4)} \times \omega_2$ ..... (8)

Angular velocity of link 4, $\omega_4 = \dfrac{b\,\sin(\theta_2-\theta_4)}{d\,\sin\,(\theta_3-\theta_4)} \times \omega_2$ ..... (9)

Differentiating the equation (4) with respect to time, we get

$$be^{i\theta_2}\left[i\frac{d\omega_2}{dt}-\omega_2^2\right]+ce^{i\theta_3}\left[i\frac{d\omega_3}{dt}-\omega_3^2\right]+de^{i\theta_4}\left[i\frac{d\omega_4}{dt}-\omega_4^2\right]=0 \quad \text{..... (10)}$$

Let $\dfrac{d\omega_2}{dt} = \alpha_2$, $\dfrac{d\omega_3}{dt} = \alpha_3$, and $\dfrac{d\omega_4}{dt} = \alpha_4$

∴ Equation (10) becomes

$be^{i\theta_2}(i\alpha_2-\omega_2^2)+ce^{i\theta_3}(i\alpha_3-\omega_3^2)+de^{i\theta_4}(i\alpha_4-\omega_4^2)=0$ ..... (11)

Substituting $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (11), we get

$b(\cos\theta_2 + i\sin\theta_2)(i\,\alpha_2-\omega_2^2)+c(\cos\theta_3+i\sin\theta_3)(i\,\alpha_3-\omega_3^2)+$

$d(\cos\theta_4+i\sin\theta_4)(i\,\alpha_4-\omega_4^2)=0$ ..... (12)

The real part of the equation (12) is

$$R_a = -b(\omega_2^2\cos\theta_2+\alpha_2\sin\theta_2)-c(\omega_3^2\cos\theta_3+\alpha_3\sin\theta_3)$$
$$-d(\omega_4^2\cos\theta_4+\alpha_4\sin\theta_4)=0 \quad \text{..... (13)}$$

The imaginary part of the equation (12) is

$$I_a = b(\alpha_2\cos\theta_2-\omega_2^2\sin\theta_2)+c(\alpha_3\cos\theta_3-\omega_3^2\sin\theta_3)$$
$$+d(\alpha_4\cos\theta_4-\omega_4^2\sin\theta_4)=0 \quad \text{..... (14)}$$

Solving the simultaneous equations (13) and (14), we get,

Angular acceleration of link 3 is,

$$\therefore \quad \alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 - \frac{b\omega_2^2\cos(\theta_2-\theta_4)+c\,\omega_3^2\cos(\theta_3-\theta_4)+d\omega_4^2}{c\,\sin(\theta_3-\theta_4)} \quad \text{..... (15)}$$

Angular acceleration of link 4 is,

$$\alpha_4 = \frac{\omega_4}{\omega_2}\alpha_2 + \frac{b\omega_2^2\cos(\theta_2-\theta_3)+c\,\omega_3^2+d\omega_4^2\cos(\theta_3-\theta_4)}{d\,\sin(\theta_3-\theta_4)} \quad \text{..... (16)}$$

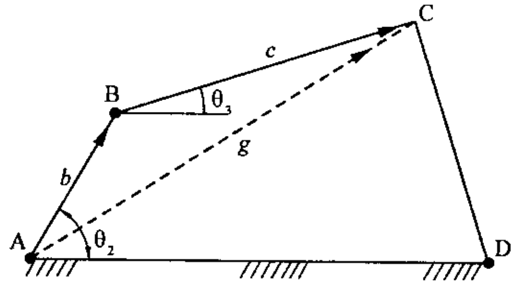

Fig. 5.2c

Having found the angular velocities of links 3 and 4, we can compute the linear velocity at any point on the links as follows. Let C be a point whose velocity is to be found (refer fig. 5.2*c*). Join AC. The position vector $\bar{g}$ is then

$$\bar{g} = \bar{b} + \bar{c} = be^{i\theta 2} + ce^{i\theta 3} \quad ..... (17)$$

Differentiate the equation (17) with respect to time,

Velocity $V_c = \dot{g} = i\, b\omega_2\, e^{i\theta 2} + i\, c\omega_3\, e^{i\theta 3}$ ..... (18)

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in equation (18), we get

$$= i\, b\omega_2 (\cos\theta_2 + i\sin\theta_2) + i\, c\omega_3 (\cos\theta_3 + i\sin\theta_3)$$

The real and imaginary components are;

$$R_v = -b\omega_2 \sin\theta_2 - c\omega_3 \sin\theta_3$$

$$I_v = b\omega_2 \cos\theta_2 + c\omega_3 \cos\theta$$

The magnitude of velocity $V_c = \sqrt{R_v^2 + I_v^2}$

and direction is, $\theta_c = \tan^{-1}\left(\frac{I_v}{R_v}\right)$

The linear acceleration of the point C on link 3 is obtained by differentiating the equation (18) with respect to time.

$\therefore$ Acceleration $A_c = \ddot{g} = be^{i\theta 2}(i\alpha_2 - \omega_2^2) + ce^{i\theta 3}(i\alpha_3 - \omega_3^2)$ ..... (19)

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in the above equation,

$$A_c = b(\cos\theta_2 + i\sin\theta_2)(i\alpha_2 - \omega_2^2) + c(\cos\theta_3 + i\sin\theta_3)(i\alpha_3 - \omega_3^2)$$

The real and imaginary components are;

$$R_a = -b(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - c(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3)$$

$$I_a = b(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) + c(\alpha_3\cos\theta_3 - \omega_3^2\sin\theta_3)$$

Acceleration $A_c = \sqrt{R_a^2 + I_a^2}$ and $\theta_c = \tan^{-1}\left(\frac{I_a}{R_a}\right)$

Example 5.1

A four far chain mechanism ABCD is made up of four links, pin jointed at the ends. AD is a fixed link which is 120 mm long. The links AB, BC and CD are 60 mm, 80 mm, and 80 mm long respectively. At certain instant, the link AB makes an angle of 60° with the link AD. If the link AB rotates at uniform speed of 10 rpm clockwise direction, determine analytically

(i) Angular velocity of the link BC and CD

(ii) Angular acceleration of the link BC and CD

Data:

AD = a = 120 mm AB = b = 60 mm, BC = c = 80 mm, CD = d = 80 mm,

$\theta_2 = 60°$, $n_2 = -10$ rpm (clockwise)

Solution:

Draw the configuration diagram as shown in fig. 5.3. We have to determine first the angles θ_3 and θ_4 made by the links 3 and 4.

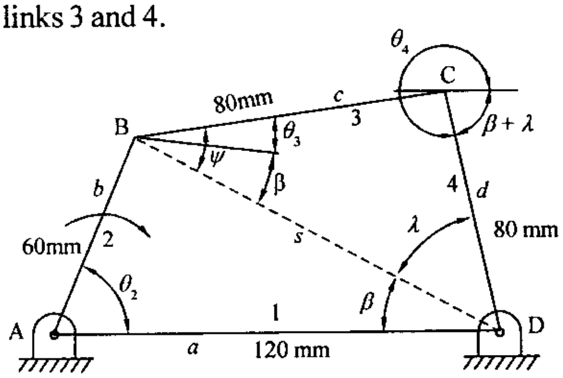

Fig. 5.3

Consider the triangle ABD,

$$\text{BD} = s = \sqrt{a^2 + b^2 - 2ab\ \cos\theta_2}$$

$$= \sqrt{120^2 + 60^2 - 2 \times 120 \times 60\ \cos 60} = 103.923 \text{ mm}$$

By law of sines, $\dfrac{\sin\ \beta}{b} = \dfrac{\sin\ \theta_2}{s}$

i.e., $\dfrac{\sin\ \beta}{60} = \dfrac{\sin\ 60}{103.923}$

∴ Angle $\beta = 30°$

From triangle BCD,

$$\text{CD} = d = \sqrt{c^2 + s^2 - 2cs\ \cos\ \psi}$$

$$\text{i.e. } 80 = \sqrt{80^2 + 103.923^2 - 2 \times 80 \times 103.923\ \cos\ \psi}$$

Squaring both sides, we get

$$80^2 = 80^2 + 103.923^2 - 2 \times 80 \times 103.923 \cos \psi$$

$\therefore$ Angle $\psi = 49.49°$

The angle made by the link BC, $\theta_3 = \psi - \beta = 49.49 - 30 = 19.49°$

Similarly, $\dfrac{\sin \lambda}{c} = \dfrac{\sin \psi}{d}$

i.e. $\dfrac{\sin \lambda}{80} = \dfrac{\sin 49.49}{80}$

$\therefore$ Angle $\lambda = 49.49°$

The angle made by the link CD, $\theta_4 = 360 - (\beta + \lambda)$

$$= 360 - (30 + 49.49) = 280.50°$$

Angular velocity of link AB, $\omega_2 = \dfrac{2\pi n_2}{60}$

$$= \frac{2\pi \times (-10)}{60} = -1.047 \text{ rad/s}$$

Angular velocity of link BC, $\omega_3 = -\dfrac{b \sin (\theta_2 - \theta_4)}{c \sin (\theta_3 - \theta_4)} \times \omega_2$

$$= \frac{-60 \sin (60 - 280.5)}{80 \sin (19.49 - 280.5)} \times (-1.047)$$

$$= 0.5163 \text{ rad/s}$$

Angular velocity of link CD, $\omega_4 = \dfrac{b \sin (\theta_2 - \theta_3)}{d \sin (\theta_3 - \theta_4)} \times \omega_2$

$$= \frac{60 \sin (60 - 19.49)}{80 \sin (19.49 - 280.5)} \times (-1.047)$$

$$= -0.5164 \text{ rad/s}$$

Angular acceleration of the link BC,

$$\alpha_3 = \frac{\omega_3}{\omega_2} \alpha_2 - \frac{b\,\omega_2^2 \cos (\theta_2 - \theta_4) + c\omega_3^2 \cos(\theta_3 - \theta_4) + d\omega_4^2}{c \sin(\theta_3 - \theta_4)}$$

Since there is no acceleration for the link AB, $\alpha_2 = 0$

$$\therefore \alpha_3 = -\frac{60 \times (-1.047)^2 \cos(60 - 280.5) + 80 \times 0.5163^2 \cos(19.49 - 280.5) + 80 \times (-0.5164)^2}{80 \times \sin(19.49 - 280.5)}$$

$$= 0.8608 \text{ rad/s}^2$$

Angular acceleration of the link CD,

$$\alpha_4 = \frac{\omega_4}{\omega_2}\alpha_2 + \frac{b\,\omega_2^2\cos(\theta_2-\theta_3)+c\,\omega_3^2+d\,\omega_4^2\cos(\theta_3-\theta_4)}{d\sin(\theta_3-\theta_4)}$$

$$= \frac{60\times(-1.047)^2\cos(60-19.49)+80\times0.5163^3+80\times(-0.5164)^2\cos(19.49-280.5)}{80\times\sin(19.49-280.5)}$$

$$= 0.8608 \text{ rad/s}^2$$

Example 5.2

In a four bar mechanism ABCD, link AB = 300 mm, BC = 360 mm, CD = 360 mm and the fixed link AD = 600 mm. The angle made by the link AB with fixed link AD is 60°. The link AB has an angular velocity of 10 rad/s and angular acceleration of 30 rad/s² both clockwise. Determine the angular velocity and angular acceleration of link BC and CD by Raven's approach.

(VTU, Feb. 2005)

Data:

AD = a = 600 mm AB = b = 300 mm, BC = c = 360 mm, CD = d = 360 mm,

$\theta_2 = 60°$, $\omega_2 = -10$ rad/s (clockwise), $\alpha_2 = 30$ rad/s² (clockwise)

Solution:

Draw the configuration diagram as shown in fig. 5.4. We have to determine first the angles θ_3 and θ_4 made by the links 3 and 4.

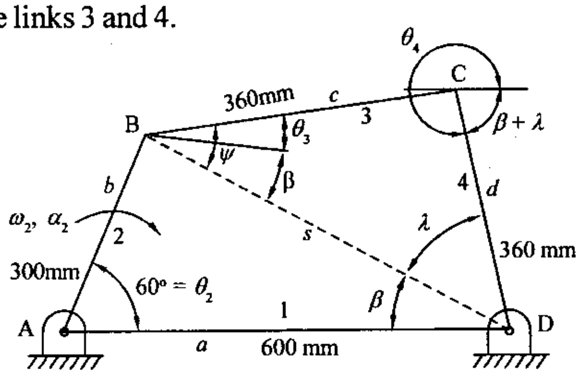

Fig. 5.4

Consider the triangle ABD,

Length BD $= s = \sqrt{a^2+b^2-2ab\cos\theta_2}$

$= \sqrt{600^2+300^2-2\times600\times300\cos 60} = 519.62$ mm

By law of sines, $\frac{\sin\beta}{b} = \frac{\sin\theta_2}{s}$

i.e., $\frac{\sin\beta}{300} = \frac{\sin 60}{519.62}$

$\therefore$ Angle $\beta = 30°$

From triangle BCD,

Length CD $= d = \sqrt{c^2 + s^2 - 2cs\cos\psi}$

i.e., $360 = \sqrt{360^2 + 519.62^2 - 2 \times 360 \times 519.62 \cos\psi}$

Squaring both sides, we get

$$360^2 = 360^2 + 519.62^2 - 2 \times 360 \times 519.62 \cos\psi$$

$\therefore$ Angle $\psi = 43.81°$

The angle made by the link BC, $\theta_3 = \psi - \beta = 43.81 - 30 = 13.81°$

Similarly, $\dfrac{\sin\lambda}{c} = \dfrac{\sin\psi}{d}$

i.e. $\dfrac{\sin\lambda}{360} = \dfrac{\sin 43.81}{360}$

$\therefore$ Angle $\lambda = 43.81°$

The angle made by the link CD, $\theta_4 = 360 - (\beta + \lambda)$

$= 360 - (30 + 43.81) = 286.19°$

Angular velocity of link AB, $\omega_2 = -10$ rad/s

Angular velocity of link BC, $\omega_3 = -\dfrac{b\sin(\theta_2 - \theta_4)}{c\sin(\theta_3 - \theta_4)} \times \omega_2$

$= \dfrac{-300\sin(60 - 286.19)}{360\sin(13.81 - 286.19)} \times (-10)$

$= 6.0188$ rad/s

Angular velocity of link CD, $\omega_4 = \dfrac{b\sin(\theta_2 - \theta_3)}{d\sin(\theta_3 - \theta_4)} \times \omega_2$

$= \dfrac{300\sin(60 - 13.81)}{360\sin(13.81 - 286.19)} \times (-10)$

$= -6.0188$ rad/s

Angular acceleration of the link BC,

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 - \frac{b\,\omega_2^2\cos(\theta_2 - \theta_4) + c\omega_3^2\cos(\theta_3 - \theta_4) + d\omega_4^2}{c\sin(\theta_3 - \theta_4)}$$

$$\therefore \alpha_3 = \frac{6.0188}{-10} \times (-30)$$

$$- \frac{300 \times 10^2\cos(60 - 286.19) + 360 \times 6.0188^2\cos(13.81 - 286.19) + 360 \times 6.0188^2}{360\sin(13.81 - 286.19)}$$

$= 38.032$ rad/s^2

Angular acceleration of the link CD,

$$\alpha_4 = \frac{\omega_4}{\omega_2}\alpha_2 + \frac{b\,\omega_2^2\cos(\theta_2-\theta_3)+c\,\omega_3^2+d\,\omega_4^2\cos(\theta_3-\theta_4)}{d\sin(\theta_3-\theta_4)}$$

$$= \frac{-6.0188}{-10}\times(-30) + \frac{300\times10^2\cos(60-13.81)+360\times6.0188^2+360\times6.0188^2\cos(13.81-286.19)}{360\sin(13.81-286.19)}$$

$$= 77.445 \text{ rad/s}^2$$

Example 5.3

Fig. 5.5a shows a four bar mechanism. Crank O_2A rotates at 100 rpm clockwise and an angular acceleration of 120 rad/s^2 clockwise at the instant when the crank makes an angle of 60° to the horizontal. Determine the angular velocities and angular accelerations of links 3 and 4 by Raven's method. Also find the velocities and acceleration of points C and D by assuming AC is parallel to $O_2\,O_4$.

Data:

$O_2O_4 = a = 250$mm, $O_2A = b = 90$ mm, AB $= c = 180$ mm, $BO_4 = d = 120 + 60 = 180$ mm, $\theta_2 = 60°$, $\theta_1 = 0$, $n_2 = -100$ rpm (clockwise), $\alpha_2 = -120$ rad/s^2 (clockwise)

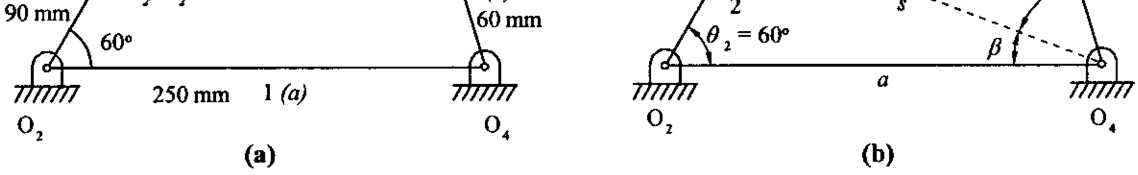

Fig. 5.5

Solution:

We have to determine the angle made by the links 3 and 4 with the horizontal.

Consider the triangle $O_2\,A\,O_4$ (Refer fig. 5.5 *b*)

$$AO_4 = s = \sqrt{a^2+b^2-2ab\cos\theta_2}$$

$$= \sqrt{250^2+90^2-2\times250\times90\cos 60} = 219.32 \text{ mm}$$

By law of sines, $\dfrac{\sin\beta}{b} = \dfrac{\sin\theta_2}{s}$

i.e., $\frac{\sin\beta}{90} = \frac{\sin 60}{219.32}$

$\therefore$ Angle $\beta = 20.82°$

From triangle ABO_4,

$$BO_4 = d = \sqrt{c^2 + s^2 - 2cs\cos\psi}$$

i.e., $180^2 = 180^2 + 219.32^2 - 2 \times 180 \times 219.32 \cos\psi$

$\therefore$ Angle $\psi = 52.47°$

Angle made by the link AB, $\theta_3 = \psi - \beta$

$= 52.47 - 20.82 = 31.65°$

Similarly, $\frac{\sin\lambda}{c} = \frac{\sin\psi}{d}$

i.e., $\frac{\sin\lambda}{180} = \frac{\sin 52.47}{180}$

$\therefore$ Angle $\lambda = 52.47°$

Angle made by link BO_4, $\theta_4 = 360 - (\beta + \lambda)$

$= 360 - (20.82 + 52.47) = 286.71°$

Angular velocity of link O_2A, $\omega_2 = \frac{2\pi n_2}{60}$

$= \frac{2\pi \times (-100)}{60} = -10.47$ rad/s

Angular velocity of link AB, $\omega_3 = \frac{-b\sin(\theta_2 - \theta_4)}{c\sin(\theta_3 - \theta_4)} \times \omega_2$

$= \frac{-90\sin(60 - 286.71)}{180\sin(31.65 - 286.71)} \times (-10.47)$

$= 3.944$ rad/s

Angular velocity of link BO_4, $\omega_4 = \frac{b\sin(\theta_2 - \theta_3)}{d\sin(\theta_3 - \theta_4)} \times \omega_2$

$= \frac{90\sin(60 - 31.65)}{180\sin(31.65 - 286.71)} \times (-10.47)$

$= -2.573$ rad/s

Angular acceleration of link AB,

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 - \frac{b\omega_2^2\cos(\theta_2-\theta_4)+c\omega_3^2\cos(\theta_3-\theta_4)+d\omega_4^2}{c\sin(\theta_3-\theta_4)}$$

$$= \frac{3.944}{-10.47}\times(-120) - \frac{90\times10.47^2\cos(60-286.71)}{180\times\sin(31.65-286.71)}$$

$$+\frac{180\times3.944^2\cos(31.65-286.71)+180\times2.573^2}{180\times\sin(31.65-286.71)}$$

$$= 81.4\text{ rad/s}^2$$

Angular acceleration of link BO_4

$$\alpha_4 = \frac{\omega_4}{\omega_2}\alpha_2 + \frac{b\omega_2^2\cos(\theta_2-\theta_3)+c\omega_3^2+d\omega_4^2\cos(\theta_3-\theta_4)}{d\sin(\theta_3-\theta_4)}$$

$$= \frac{-2.573}{-10.47}\times(-120) + \frac{90\times10.47^2\cos(60-31.65)+180\times3.944^2}{180\times\sin(31.65-286.71)}$$

$$+\frac{180\times2.573^2\cos(31.65-286.71)}{180\times\sin(31.65-286.71)} = 34.767\text{ rad/s}^2$$

Velocity and acceleration of point C :

Join O_2 C by straight line as shown in fig. 5.5c. The loop-closure equation for the triangle O_2AC is

$$\bar{p} = \bar{b} + \bar{q}$$

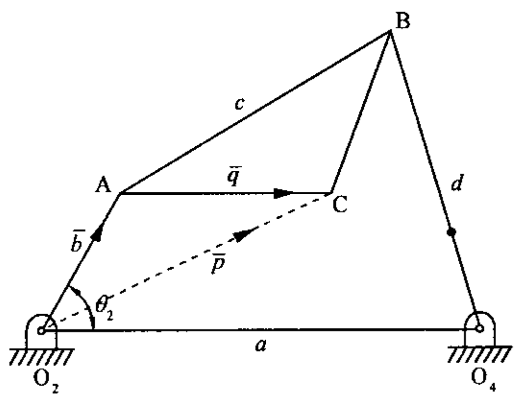

Fig. 5.5c

Expressing the above equation in exponential form, we get

$$\bar{p} = be^{i\theta 2} + qe^{i\theta 5} \qquad .....(1)$$

where θ_2 is the angle made by the link O_2A with horizontal = 60° and θ_5 is the angle made by AC with horizontal = 0°

(since AC is parallel to $O_2 O_4$)

Differentiating the equation (1) with respect to time, we get

$$\text{Velocity of point C, } V_c = ib\frac{d\theta_2}{dt}e^{i\theta_2} + iq\frac{d\theta_5}{dt}e^{i\theta_5} \quad \text{..... (2)}$$

$$\text{Let} \quad \frac{d\theta_2}{dt} = \omega_2, \quad \text{and} \quad \frac{d\theta_5}{dt} = \omega_5 = \omega_3$$

∴ Equation (2) becomes,

$$V_c = ib\,\omega_2\,e^{i\theta_2} + iq\,\omega_3\,e^{i\theta_5} \quad \text{..... (3)}$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (3), we get

$$V_c = ib\omega_2(\cos\theta_2 + i\sin\theta_2) + iq\omega_3(\cos\theta_5 + i\sin\theta_5) \quad \text{..... (4)}$$

Real part of the equation (4) is

$$R_v = -b\omega_2\sin\theta_2 - q\omega_3\sin\theta_5$$

i.e., $R_v = -90 \times (-10.47)\sin 60 - 120 \times 3.944 \sin 0 = 816.06$

Imaginary part of the equation (4) is

$$I_v = b\omega_2\cos\theta_2 + q\omega_3\cos\theta_5$$

i.e., $I_v = 90 \times (-10.47)\cos 60 + 120 \times 3.944\cos 0 = 2.13$

$$\text{Velocity of C, } V_c = \sqrt{R_v^2 + I_v^2}$$

$$= \sqrt{816.06^2 + 2.13^2} = 816.06 \text{ mm/s} = 0.816 \text{ m/s}$$

Direction of V_c is

$$\phi_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\frac{2.13}{816.06} = 0.15°$$

The linear acceleration of point C is obtained by differentiating equation (3) with respect to time.

$$\therefore \quad A_c = be^{i\theta_2}\left(i\frac{d\omega_2}{dt} - \omega_2^2\right) + qe^{i\theta_5}\left(i\frac{d\omega_3}{dt} - \omega_3^2\right) \quad \text{..... (5)}$$

$$\text{Let} \quad \frac{d\omega_2}{dt} = \alpha_2 \quad \text{and} \quad \frac{d\omega_3}{dt} = \alpha_3$$

∴ Equation (5) becomes.

$$A_c = be^{i\theta_2}(i\alpha_2 - \omega_2^2) + qe^{i\theta_5}(i\alpha_3 - \omega_3^2) \quad \text{..... (6)}$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (6), we get

$$A_c = b(\cos\theta_2 + i\sin\theta_2)(i\alpha_2 - \omega_2^2) + q(\cos\theta_5 + i\sin\theta_5)(i\alpha_3 - \omega_3^2) \quad \text{.... (7)}$$

Real part of the equation (7) is

$$R_a = -b(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - q(\omega_3^2\cos\theta_5 + \alpha_3\sin\theta_5)$$

i.e. $$R_a = -90(10.47^2\cos 60 - 120\sin 60) - 120(3.944^2\cos 0 + 81.4\sin 0)$$

$$= 2553.52$$

Imaginary part of the equation (7) is

$$I_a = b(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) - q(\alpha_3\cos\theta_5 - \omega_3^2\sin\theta_5)$$

i.e. $$I_a = 90(-120\cos 60 - 10.47^2\sin 60) + 120(81.4\cos 0 - 3.944^2\sin 0)$$

$$= -4176.1$$

Acceleration of C, $A_c = \sqrt{R_a^2 + I_a^2}$

$$= \sqrt{2553.52^2 + 4176.1^2} = 4894.9 \text{ mm/s}^2 = 4.895 \text{ m/s}^2$$

Direction of A_c, $\phi_A = \tan^{-1}\left(\frac{I_a}{R_a}\right)$

$$= \tan^{-1}\left(\frac{-4176.1}{2553.5}\right) = 360 - \tan^{-1}\left(\frac{4176.1}{2553.5}\right) = 301.4^\circ$$

Velocity and acceleration of point D :

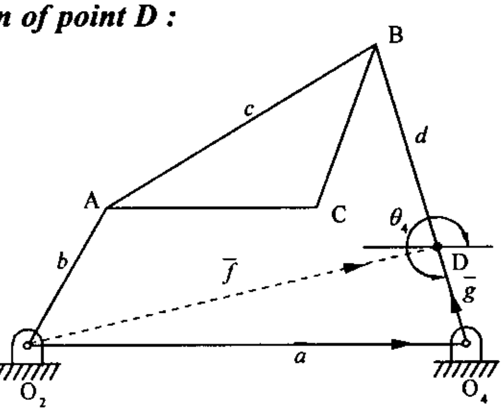

Fig. 5.5 *d*

Join O_2 D by straight line as shown in fig 5.5d. The loop-closure equation for the triangle O_2 D O_4 is

$$\bar{f} = \bar{a} + \bar{g}$$

Expressing the above equation in exponential form, we get

$$\bar{f} = ae^{i\theta_1} + ge^{i\theta_4}$$

where θ_1 is the angle made by the link O_2O_4 with horizontal = 0°

θ_4 is the angle made by O_4 D with horizontal = 286.71°

The above equation can be written as

$$\bar{f} = a + ge^{i\theta_4} \qquad (\because \theta_1 = 0°) \qquad \text{..... (1)}$$

(Note that $O_2\,O_4$ is a fixed link)

Differentiating the equation (1) with respect to time, we get

Velocity of point D, $$V_D = ig\frac{d\theta_4}{dt}e^{i\theta_4} \qquad \text{..... (2)}$$

Let $$\frac{d\theta_4}{dt} = \omega_4$$

$$\therefore \qquad V_D = ig\omega_4 e^{i\theta_4} \qquad \text{..... (3)}$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (3), we get

$$V_D = ig\,\omega_4(\cos\theta_4 + i\sin\theta_4) \qquad \text{..... (4)}$$

Real part of the equation (4) is

$$R_v = -g\omega_4\sin\theta_4$$

$$= -60 \times (-2.573)\sin 286.71 = -147.86$$

Imaginary part of the equation (4) is

$$I_v = g\omega_4\cos\theta_4$$

$$= 60 \times (-2.573)\cos 286.71 = -44.388$$

$\therefore$ Velocity of D, $$V_D = \sqrt{R_v^2 + I_v^2}$$

$$= \sqrt{147.86^2 + 44.388^2} = 154.4 \text{ mm/s} = 0.1544 \text{ m/s}$$

Direction of V_D, $$\phi_V = \tan^{-1}\left(\frac{I_v}{R_v}\right)$$

$$= \tan^{-1}\left(\frac{-44.388}{-147.86}\right) = 180 + \tan^{-1}\left(\frac{44.388}{147.86}\right) = 196.71°$$

The linear acceleration of point D is obtained by differentiating the equation (3) with respect to time.

$$\therefore \qquad A_D = ge^{i\theta_4}\left(i\frac{d\omega_4}{dt} - \omega_4^2\right) \qquad \text{..... (5)}$$

Let $$\frac{d\omega_4}{dt} = \alpha_4$$

$\therefore$ Equation (5) becomes,

$$A_D = ge^{i\theta_4}(i\,\alpha_4 - \omega_4^2) \qquad .....(6)$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (6), we get

$$A_D = g(\cos\theta_4 + i\sin\theta_4)(i\,\alpha_4 - \omega_4^2) \qquad .....(7)$$

Real part of the equation (7) is

$$R_a = -g(\omega_4^2\cos\theta_4 + \alpha_4\sin\theta_4)$$

i.e., $R_a = -60\,(2.573^2\cos 286.71 + 34.764\sin 286.71) = 1883.6$

Imaginary part of the equation (7) is

$$I_a = g(\alpha_4\cos\theta_4 - \omega_4^2\sin\theta_4)$$

i.e., $I_a = 60\,(34.767\cos 286.71 - 2.573^2\sin 286.71) = 980.24$

Acceleration of D, $A_D = \sqrt{R_a^2 + I_a^2}$

$$= \sqrt{1883.6^2 + 980.24^2} = 2123.4 \text{ mm/s} = 2.1234 \text{ m/s}^2$$

Direction of A_D, $\phi_A = \tan^{-1}\left(\frac{I_a}{R_a}\right) = \tan^{-1}\left(\frac{980.24}{1883.6}\right) = 27.5^\circ$

Example 5.4

A four bar chain has a fixed link AD = 1 m, driving crank AB = 0.3 m, follower link CD = 0.6 m and the connecting link BC = 1.2 m. Using Raven's approach determine the acceleration of point P midway between B and C, when the angle BAD = 135° and the crank AB rotates at a speed of 300 rpm clockwise with an angular acceleration of 200 rad/s² in anti-clockwise direction.

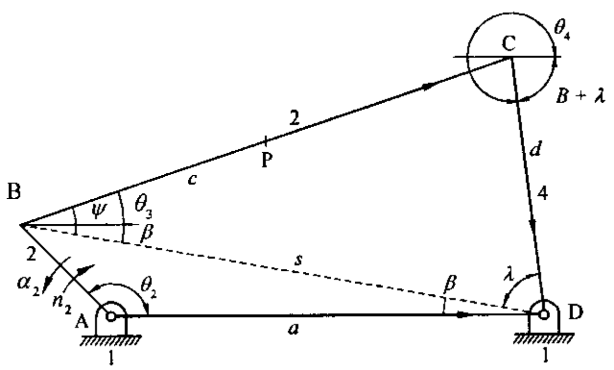

Fig. 5.6a

Data :

$a = 1$ m, $b = 0.3$ m, $c = 1.2$ m, $d = 0.6$ m, $\theta_2 = 135°$, $n_2 = -300$ rpm (clockwise), $\alpha_2 = 200$ rad/s² (anti-clockwise)

Solution :

First we have to determine the angles θ_3 and θ_4 made by the links 3 and 4 respectively with the horizontal.

Consider the triangle ABD (refer fig. 5.6*a*)

$$BD = s = \sqrt{a^2 + b^2 - 2ab\cos\theta_2}$$

$$= \sqrt{1^2 + 0.3^2 - 2 \times 1 \times 0.3 \cos 135} = 1.2306 \text{ m}$$

By law of sines, $$\frac{\sin\beta}{b} = \frac{\sin\theta_2}{s}$$

i.e., $$\frac{\sin\beta}{0.3} = \frac{\sin 135}{1.2306}$$

$\therefore$ Angle $\beta = 9.926°$

From triangle BCD, $$CD = d = \sqrt{c^2 + s^2 - 2cs\cos\psi}$$

i.e., $$0.6 = \sqrt{1.2^2 + 1.2306^2 - 2 \times 1.2 \times 1.2306 \cos\psi}$$

Squaring both sides, we get $0.6^2 = 1.2^2 + 1.2306^2 - 2 \times 1.2 \times 1.2306 \cos\psi$

$\therefore$ Angle $\psi = 28.547°$

Angle made by the link BC, $\theta_3 = \psi - \beta = 28.547 - 9.926 = 18.621°$

Also, $$\frac{\sin\lambda}{c} = \frac{\sin\psi}{d}$$

i.e., $$\frac{\sin\lambda}{1.2} = \frac{\sin 28.547}{0.6}$$

$\therefore$ Angle $\lambda = 72.893°$

Angle made by the link CD, $\theta_4 = 360 - (\beta + \lambda)$

$$= 360 - (9.926 + 72.893) = 277.181°$$

Angular velocity of link AB, $$\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times (-300)}{60} = -31.416 \text{ rad/s}$$

Angular velocity of link BC, $$\omega_3 = -\frac{b\sin(\theta_2 - \theta_4)}{c\sin(\theta_3 - \theta_4)} \times \omega_2$$

$$= -\frac{0.3 \times \sin(135 - 277.181)}{1.2 \times \sin(18.621 - 277.181)} \times (-31.416)$$

$$= -4.913 \text{ rad/s}$$

Angular velocity of link CD, $\omega_4 = \dfrac{b\sin(\theta_2-\theta_3)}{d\sin(\theta_3-\theta_4)}\times\omega_2$

$$= \frac{0.3\times\sin(135-18.621)}{0.6\times\sin(18.621-277.181)}\times(-31.416)$$

$$= -14.358 \text{ rad/s}$$

Angular acceleration of link BC,

$$\alpha_3 = \frac{\omega_3}{\omega_2}\,\alpha_2 - \frac{b\omega_2^2\cos(\theta_2-\theta_4)+c\,\omega_3^2\cos(\theta_3-\theta_4)+d\,\omega_4^2}{c\sin(\theta_3-\theta_4)}$$

$$= \frac{-4.913}{-31.416}\times 200 - \frac{0.3\times 31.416^2\cos(135-277.181)+1.2\times 4.913^2\cos(18.621-277.181)+0.6\times 14.358^2}{1.2\times\sin(18.621-277.181}$$

$= 129.86$ rad/s^2

Angular acceleration of link CD,

$$\alpha_4 = \frac{\omega_4}{\omega_2}\,\alpha_2 + \frac{b\omega_2^2\cos(\theta_2-\theta_3)+c\omega_3^2+d\omega_4^2\cos(\theta_3-\theta_4)}{d\sin(\theta_3-\theta_4)}$$

$$= \frac{-14.358}{-31.416}\times 200 + \frac{0.3\times 31.416^2\cos(135-18.621)+1.2\times 4.913^2+0.6\times 14.358^2\cos(18.621-277.181)}{0.6\times\sin(18.621-277.181)}$$

$= 124.76$ rad/s^2

***Velocity and acceleration of point P:** (Refer fig. 5.6b)*

Length $\quad BP = f = \dfrac{BC}{2} = \dfrac{1.2}{2} = 0.6$ m

Join AP by straight line as shown in fig. 5.6*b*

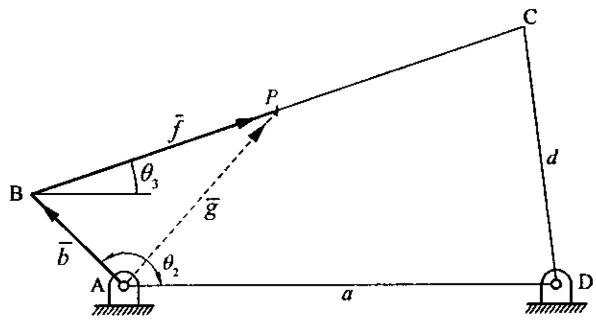

Fig. 5.6*b*

The loop-closure equation for the triangle ABP is

$$\bar{g} = \bar{b}\times\bar{f} \qquad .....(1)$$

Expressing the above equation in complex form,

$$\bar{g} = b\,e^{i\theta 2} + f\,e^{i\theta 3} \quad \text{..... (2)}$$

Differentiating the equation (2) with respect to time, we get

$$\text{Velocity of P,} \quad V_p = \dot{g} = ib\,\frac{d\theta_2}{dt}\,e^{i\theta 2} + i f\,\frac{d\theta_3}{dt}\,e^{i\theta 3} \quad \text{..... (3)}$$

$$= i\,b\omega_2\,e^{i\theta 2} + i f\,\omega_3\,e^{i\theta 3} \qquad \left(\because \frac{d\theta}{dt} = \omega\right) \quad \text{..... (4)}$$

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in equation (4), we get

$$V_p = i\,b\omega_2(\cos\theta_2 + i\sin\theta_2) + i f\omega_3(\cos\theta_3 + i\sin\theta_3) \quad \text{..... (5)}$$

Real part of the velocity equation (5) is

$$R_v = -b\,\omega_2\sin\theta_2 - f\,\omega_3\sin\theta_3$$

$$= -0.3\times(-31.416)\sin 135 - 0.6\times(-4.913)\sin 18.621 = 7.606$$

Imaginary part of the velocity equation (5) is

$$I_v = b\omega_2\cos\theta_2 + f\,\omega_3\cos\theta_3$$

$$= 0.3\times(-31.416)\cos 135 + 0.6\times(-4.913)\cos 18.621 = 3.8708$$

$$\text{Velocity of P,} \quad V_p = \sqrt{R_v^2 + I_v^2} = \sqrt{7.606^2 + 3.8708^2} = 8.534\ \text{m/s}$$

$$\text{Direction of } V_p, \quad \theta_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\left(\frac{3.8708}{7.606}\right) = 26.97^\circ$$

Differentiate the equation (3) with respect to time,

$$\text{Acceleration of P,} \quad A_p = \ddot{g} = b\,e^{i\theta 2}\left[\frac{i\,d^2\theta_2}{dt^2} - \left(\frac{d\theta_2}{dt}\right)^2\right] + f\,e^{i\theta 3}\left[\frac{i\,d^2\theta_3}{dt^2} - \left(\frac{d\theta_3}{dt}\right)^2\right]$$

$$= b\,e^{i\theta 2}(i\alpha_2 - \omega_2^2) + f\,e^{i\theta 3}(i\alpha_3 - \omega_3^2) \qquad \left(\because \frac{d\theta}{dt} = \omega \ \text{ and } \ \frac{d^2\theta}{dt^2} = \alpha\right)$$

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in the above equation, we get

$$A_p = b(\cos\theta_2 + i\sin\theta_2)(i\alpha_2 - \omega_2^2) + f(\cos\theta_3 + i\sin\theta_3)(i\alpha_3 - \omega_3^2) \quad \text{..... (6)}$$

Real part of the acceleration equation (6) is

$$R_a = -b(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - f(\omega_3^2\cos\theta_3 + \theta_3\sin\theta_3)$$

$$= -0.3(31.416^2\cos 135 + 20\sin 135) - 0.6(4.913^2\cos 18.621 + 101.71\sin 18.621) = 171.914$$

Imaginary part of the acceleration equation (6) is

$$I_a = b(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) + f(\alpha_3\cos\theta_3 - \omega_3^2\sin\theta_3)$$

$$= 0.3(20\cos 135 - 31.416^2\sin 135) + 0.6(101.71\cos 18.621 - 4.913^2\sin 18.621) = -160.4$$

Acceleration of point P, $A_p = \sqrt{R_a^2 + I_a^2} = \sqrt{171.914^2 + 160.4^2} = 235.12$ m/s

Direction of A_p, $\theta_A = \tan^{-1}\left(\frac{I_a}{R_a}\right) = \tan^{-1}\left(\frac{-160.4}{171.914}\right)$

$= 360 - \tan^{-1}\left(\frac{160.4}{171.914}\right) = 316.98°$

Single slider-crank mechanism

In the slider-crank mechanism shown in fig. 5.7*a*, the crank 2 rotates at constant angular velocity ω_2 in the anti-clockwise direction. The velocity V_p and the acceleration A_p of the slider are to be determined. Let *b* and *c* are the fixed lengths of links 2 and 3, and *a* is a variable length giving the position of the slider *P*. The angle θ_2 is the angular position of the crank. The position of *P* relative to fixed point O is given by the vector $\bar{a}$ *(refer fig. 5.7b)*.

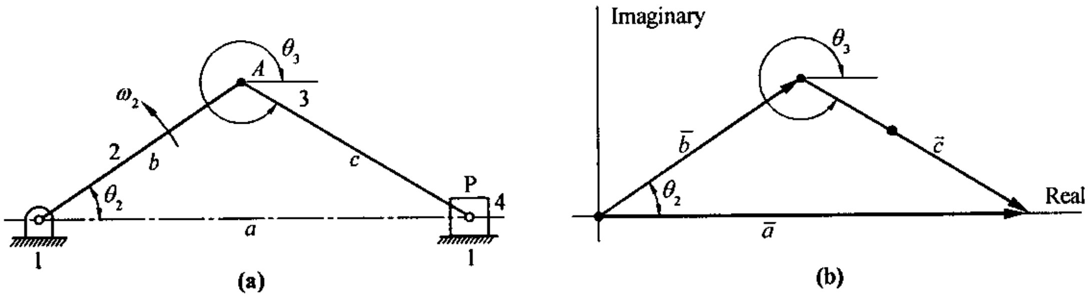

Fig. 5.7

The loop-closure or vector equation is

$$\bar{a} = \bar{b} + \bar{c} \quad \text{..... (1)}$$

Expressing the vector in complex form, we get

$$ae^{i\theta 1} = be^{i\theta 2} + ce^{i\theta 3} \quad \text{..... (2)}$$

Since the angle $\theta_1 = 0°$, so that $e^{i\theta 1} = 1$

$$\therefore \quad a = be^{i\theta 2} + ce^{i\theta 3} \quad \text{..... (3)}$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (3), we get

$$a = b(\cos\theta_2 + i\sin\theta_2) + c(\cos\theta_3 + i\sin\theta_3) \quad \text{..... (4)}$$

Separating the real and imaginary parts of the equation (4),

Real part $\quad R_S = a = b\cos\theta_2 + c\cos\theta_3 \quad$ ..... (5)

Imaginary part $\quad I_S = b\sin\theta_2 + c\sin\theta_3 \quad$ ..... (6)

Since the position vector *a* lies on the real (horizontal) axis, the imaginary part is equal to zero.

$\therefore \quad I_S = b \sin \theta_2 + c \sin \theta_3 = 0$

i.e., $\sin \theta_3 = \dfrac{-b}{c} \sin \theta_2$

or $\theta_3 = 360 - \sin^{-1}\left(\dfrac{b}{c}\sin\theta_2\right)$ ..... (7)

Equation (7) can be used to determine the angle θ_3 made by the link 3 with the horizontal. Knowing the value of θ_3, equation (5) can be used to determine a, the position of the slider from the crank shaft center.

Differentiate the equation (3) with respect to time, we get

Velocity $V_p = \dot{a} = ib \dfrac{d\theta_2}{dt} e^{i\theta 2} + ic \dfrac{d\theta_3}{dt} e^{i\theta 3}$ ..... (8)

$= ib\, \omega_2 e^{i\theta 2} + ic\, \omega_3 e^{i\theta 3} \left(\because \dfrac{d\theta_2}{dt} = \omega_2 \text{ and } \dfrac{d\theta_3}{dt} = \omega_3\right)$ ..... (9)

Substitute $e^{i\theta} = \cos \theta + i \sin \theta$ in equation (9), we get

$V_p = \dot{a} = ib\, \omega_2 (\cos \theta_2 + i \sin \theta_2) + ic\, \omega_3 (\cos \theta_3 + i \sin \theta_3)$ ..... (10)

Separating the real and imaginary parts of the equation (10), we get

Real part $R_v = -b\, \omega_2 \sin \theta_2 - c\, \omega_3 \sin \theta_3$ ..... (11)

Imaginary part $I_v = b\, \omega_2 \cos \theta_2 + c\, \omega_3 \cos \theta_3$ ..... (12)

Since the velocity vector of the slider lies on the real (horizontal) axis, the imaginary part is zero.

i.e., $I_v = b\, \omega_2 \cos \theta_2 + c\, \omega_3 \cos \theta_3 = 0$ ..... (13)

$\therefore$ Angular velocity of link 3, $\omega_3 = -\dfrac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$ ..... (14)

Velocity of the slider $V_p = \sqrt{R_v^2 + I_v^2} = R_v = -b\, \omega_2 \sin \theta_2 - c\omega_3 \sin \theta_3$ $\quad (\because\ I_v = 0)$

Differentiating the equation (8) with respect to time, we get

Acceleration

$$A_p = \ddot{a}$$

$$= ib\left[\frac{d^2\theta_2}{dt^2} e^{i\theta_2} + \frac{d\theta_2}{dt} ie^{i\theta_2} \times \frac{d\theta_2}{dt}\right] + ic\left[\frac{d^2\theta_3}{dt^3} e^{i\theta_3} + \frac{d\theta_3}{dt} ie^{i\theta_3} \times \frac{d\theta_3}{dt}\right]$$

$$= be^{i\theta 2}\left[i\frac{d^2\theta_2}{dt^2} - \left(\frac{d\theta_2}{dt}\right)^2\right] + ce^{i\theta 3}\left[i\frac{d^2\theta_3}{dt^3} - \left(\frac{d\theta_3}{dt}\right)^2\right]$$

$$= be^{i\theta 2}(i\alpha_2 - \omega_2^2) + ce^{i\theta 3}(i\alpha_3 - \omega_3^2) \left(\because \frac{d^2\theta}{dt^2} = \alpha \text{ and } \frac{d\theta}{dt} = \omega\right) \quad ...(16)$$

Substitute $e^{i\theta} = \cos\theta + i\sin\theta$ in equation (16),

$$A_p = \ddot{a} = b\,(\cos\theta_2 + i\sin\theta_2)\,(i\alpha_2 - \omega_2^2) + c\,(\cos\theta_3 + i\sin\theta_3)\,(i\alpha_3 - \omega_3^2) \qquad .....(17)$$

Real part of the equation (17) is

$$R_a = -b\,(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - c\,(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3) \qquad .....(18)$$

Imaginary part of the equation (17) is

$$I_a = b\,(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) + c\,(\alpha_3\cos\theta_3 - \omega_3^2\sin\theta_3) \qquad .....(19)$$

Since the acceleration vector of the slider lies on the real (horizontal) axis, the imaginary part is equal to zero.

i.e., $$I_a = b\,(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) + c\,(\alpha_3\cos\theta_3 - \theta_3^2\sin\theta_3) = 0$$

$\therefore$ Angular acceleration $$\alpha_3 = \frac{b\,\omega_2^2\sin\theta_2 + c\omega_3^2\sin\theta_3}{c\cos\theta_3} - \frac{b\alpha_2\cos\theta_2}{c\cos\theta_3} \qquad .....(20)$$

From equation (14), $$\frac{b\cos\theta_2}{c\cos\theta_3} = -\frac{\omega_3}{\omega_2}$$

$\therefore$ Equation (20) becomes,

$$\alpha_3 = \frac{b\omega_2^2\sin\theta_2 + c\omega_3^2\sin\theta_3}{c\cos\theta_3} + \frac{\omega_3}{\omega_2}\,\alpha_2 \qquad .....(21)$$

For constant angular velocity of the crank ω_2, the angular acceleration α_2 is zero.

$\therefore$ Angular acceleration of link 3, $$\alpha_3 = \frac{b\omega_2^2\sin\theta_2 + c\omega_3^2\sin\theta_3}{c\cos\theta_3}$$

Acceleration of the slider $$A_p = \sqrt{R_a^2 + I_a^2} = R_a \qquad (\therefore\ I_a = 0) \qquad .....(22)$$

$\therefore$ $$A_p = -b\,(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - c\,(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3) \qquad .....(23)$$

Having found the angular acceleration of link 3 (connecting rod), we can compute the linear velocity and acceleration at any point on the link as follows. Let Q be a point on the connecting rod which lies at a distance d from the crank pin (refer fig. 5.7c). Join OQ.

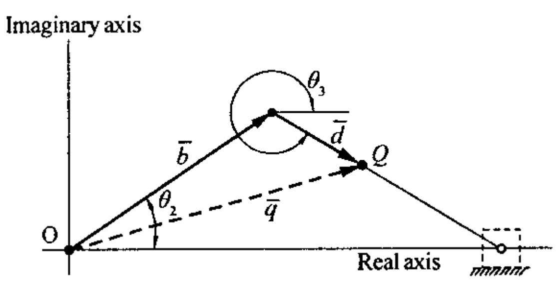

Fig. 5.7c

The loop-closure equation is, $\bar{q} = \bar{b} + \bar{d} = be^{i\theta_2} + de^{i\theta_3}$ ..... (1)

Differentiating the equation (1) with respect to time,

$$\text{Velocity} \quad V_q = ib\frac{d\theta_2}{dt}e^{i\theta_2} + id\frac{d\theta_3}{dt}e^{i\theta_3} = ib\,\omega_2 e^{i\theta_2} + id\,\omega_3 e^{i\theta_3} \qquad \text{..... (2)}$$

$$= ib\omega_2(\cos\theta_2 + i\sin\theta_2) + id\omega_3(\cos\theta_3 + i\sin\theta_3) \qquad \text{..... (3)}$$

The real and imaginary components are;

$$R_v = -b\,\omega_2\sin\theta_2 - d\,\omega_3\sin\theta_3 \qquad \text{..... (4)}$$

$$I_v = b\,\omega_2\cos\theta_2 + d\,\omega_3\cos\theta_3 \qquad \text{..... (5)}$$

Magnitude of velocity $V_q = \sqrt{R_v^2 + I_v^2}$ ..... (6)

Angle made by the velocity vector with real axis $\theta_q = \tan^{-1}\left(\frac{I_v}{R_v}\right)$ ..... (7)

The linear acceleration of Q is obtained by differentiating the equation (2) with respect to time.

$$\therefore \text{ Acceleration } A_q = be^{i\theta_2}(i\alpha_2 - \omega_2^2) + de^{i\theta_3}(i\alpha_3 - \omega_3^2) \qquad \text{..... (8)}$$

$$= b(\cos\theta_2 + i\sin\theta_2)(i\,\alpha_2 - \omega_2^2) + d(\cos\theta_3 + i\sin\theta_3)(i\,\alpha_3 - \omega_3^2)$$

The real and the imaginary components are;

$$R_a = -b(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - d(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3) \qquad \text{..... (9)}$$

$$I_a = b(\alpha_2\cos\theta_2 - \omega_2^2\sin\theta_2) + d(\alpha_3\cos\theta_3 - \omega_3^2\sin\theta_3) \qquad \text{..... (10)}$$

Magnitude of acceleration $A_q = \sqrt{R_a^2 + I_a^2}$ ..... (11)

Angle made by the acceleration vector with real axis $\theta_q = \tan^{-1}\left(\frac{I_a}{R_a}\right)$ ..... (12)

Example 5.5

The slider crank of an internal combustion engine has a crank of 150 mm length and a connecting rod of 600 mm length. The crank rotates at a constant speed of 300 rpm counterclockwise. Determine the position, velocity and acceleration of the slider when the crank angle is 45° from the inner dead center position by complex algebra.

Data:

$b = 150$ mm, $c = 600$ mm, $n_2 = 300$ rpm (counter clockwise), $\theta_2 = 45°$, $\alpha_2 = 0$ (constant crank speed)

Solution: *(refer fig. 5.8)*

$$\text{Angle made by the connecting rod } \theta_3 = 360 - \sin^{-1}\left(\frac{b}{c}\sin\theta_2\right)$$

$$= 360 - \sin^{-1}\left(\frac{150}{600}\times\sin 45\right) = 349.82°$$

Angular velocity of the crank $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times 300}{60} = 31.416$ rad/s.

Angular velocity of connecting rod $\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$

$$= \frac{-150 \times \cos 45}{600 \cos 349.82} \times 31.416 = -5.642 \text{ rad/s}$$

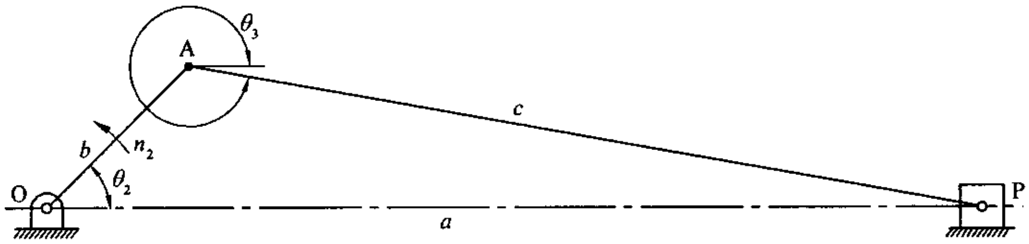

Fig. 5.8

Angular acceleration of connecting rod $\alpha_3 = \frac{b\omega_2^2 \sin\theta_2 + c\omega_3^2 \sin\theta_3}{c\cos\theta_3} + \frac{\omega_3}{\omega_2}\alpha_2$

$$= \frac{150 \times 31.416^2 \sin 45 + 600 \times (-5.642)^2 \sin 349.82}{600 \cos 349.82} + 0$$

$= 171.55$ rad/s^2

From equation (5),

Position of the slider $a = b\cos\theta_2 + c\cos\theta_3$

$= 150 \cos 45 + 600 \cos 349.82 = 696.62$ mm

i.e., The slider is at a distance of 696.62 mm from the crank shaft center

Velocity of the slider:

From equation (10),

Real part $R_v = -b\omega_2 \sin\theta_2 - c\omega_3 \sin\theta_3$

$= -150 \times 31.416 \sin 45 - 600 \times (-5.642) \sin 349.82 = -3930.5$

Imaginary part $I_v = b\omega_2 \cos\theta_2 + c\omega_3 \cos\theta_3 = 0$

Velocity of slider $V_p = \sqrt{R_v^2 + I_v^2} = \sqrt{3930.5^2 + 0} = 3930.5$ mm/s $= 3.93$ m/s

Direction $\theta_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\left(\frac{0}{-3930.5}\right)$ ($\because \theta_v$ lies in the second quadrant)

$= 180 - \tan^{-1}\left(\frac{0}{3930.5}\right) = 180°$

i.e., the direction of the velocity which lies on the line of stroke is towards the crank end.

Acceleration of the slider:

From equation (18),

Real part $R_a = -b\,(\omega_2^2 \cos\theta_2 + \alpha_2 \sin\theta_2) - c\,(\omega_3^2 \cos\theta_3 + \alpha_3 \sin\theta_3)$

$= -150\,(31.416^2 \cos 45 + 0)$

$-600\,[(-5.642)^2 \cos 349.82 + 171.55 \sin 349.82]$

$= -105290.1\ \text{mm/s}^2$

Imaginary part $I_a = b\,(\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) + c\,(\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3) = 0$

Acceleration of slider $A_p = \sqrt{R_a^2 + I_a^2}$

$= \sqrt{105290.1^2 + 0} = 105290.1\ \text{mm/s}^2 = 105.29\ \text{m/s}^2$

Direction $\theta_A = \tan^{-1}\left(\dfrac{I_a}{R_a}\right) = \tan^{-1}\left(\dfrac{0}{-105290.1}\right)$ (θ_A lies in 2nd quadrant)

$= 180 - \tan^{1}\left(\dfrac{0}{105290.1}\right) = 180^\circ$

i.e., the direction of acceleration which lies on the line of stroke is towards the crank end.

Example 5.6

Using complex algebra derive expression for velocity and acceleration of the piston and angular acceleration of connecting rod of a reciprocating engine mechanism. With these expression, determine the above quantities, if the crank length is 50 mm, connecting rod of 200 mm, constant crank speed of 3000 rpm and the crank angle of 30°. **(VTU, Feb. 2006)**

Data:

$b = 50$ mm, $c = 200$ mm, $n_2 = -3000$ rpm (assuming clockwise), $\theta_2 = 30°$, $\alpha_2 = 0$ (constant crank speed)

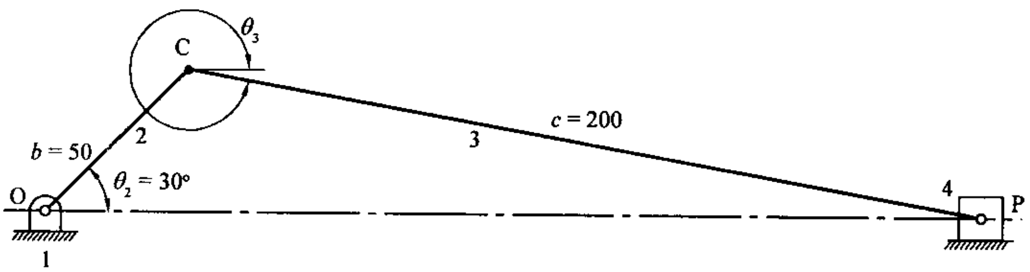

Fig. 5.9

Solution:

Draw the configuration diagram as shown in fig. 5.9. We have to determine first the angle made by the connecting rod θ_3.

Angle made by the connecting rod CP, $\theta_3 = 360 - \sin^{-1}\left(\frac{b}{c}\sin\theta_2\right)$

$$= 360 - \sin^{-1}\left(\frac{50}{200} \times \sin 30\right) = 352.82^\circ$$

Angular velocity of the crank $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times (-3000)}{60} = -314.16$ rad/s

Angular velocity of the connecting rod $\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$

$$= \frac{-50 \times \cos 30}{200 \cos 352.82} \times (-314.16) = 68.56 \text{ rad/s}$$

Angular acceleration of the connecting rod $\alpha_3 = \frac{b\omega_2^2 \sin\theta_2 + c\omega_3^2 \sin\theta_3}{c \cos\theta_3} + \frac{\omega_3}{\omega_2}\alpha_2$

$$= \frac{50 \times 314.16^2 \sin 30 + 200 \times 68.56^2 \sin 352.82}{200 \cos 352.82} + 0 = 11842.43 \text{ rad/s}^2$$

Position of the piston $a = b \cos\theta_2 + c \cos\theta_3$

$= 50 \cos 30 + 200 \cos 352.82 = 241.733$ mm

The piston is at a distance of 241.733 mm from the crank shaft center.

Velocity of the piston:

From equation (10),

Real part of $R_v = -b\omega_2 \sin\theta_2 - c\omega_3 \sin\theta_3$

$= -50 \times (-314.16) \sin 30 - 200 \times 68.56 \sin 352.82 = 9567.82$

Imaginary part $I_v = b\omega_2 \cos\theta_2 + c\omega_3 \cos\theta_3 = 0$ ($\because$ the piston moves on real axis)

Velocity of piston $V_p = \sqrt{R_v^2 + I_v^2} = \sqrt{9567.82^2 + 0} = 9567.82$ mm/s

Direction $\theta_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\left(\frac{0}{9567.82}\right) = 0^\circ$

The direction of the velocity which lies on the line of stroke is away from the crank end.

Acceleration of the slider:

From equation (18),

Real part $R_a = -b(\omega_2^2 \cos\theta_2 + \alpha_2 \sin\theta_2) - c(\omega_3^2 \cos\theta_3 + \alpha_3 \sin\theta_3)$

$= -50(314.16^2 \cos 30 + 0) - 200(68.56^2 \cos 352.82 + 11842.43 \sin 352.82)$

$= -4910377.17$ mm/s^2

Imaginary part $I_a = b(\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) + c(\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3) = 0$

Acceleration of piston $A_p = \sqrt{R_a^2 + I_a^2}$

$$= \sqrt{4910377.17^2 + 0^2} = 4910377.17 \text{ mm/s}^2$$

Direction $\theta_A = \tan^{-1}\left(\frac{I_a}{R_a}\right) = \tan^{-1}\left(\frac{0}{-4910377.17}\right)$

$$= 180 - \tan^1\left(\frac{0}{4910377.17}\right) = 180^\circ$$

i.e., the acceleration vector is towards the crank end.

Example 5.7

The crank of an engine is 200 mm. long and the ratio of connecting rod length to crank radius is 4. Determine the acceleration of the piston when the crank has turned through 45° from the inner dead center position and moving towards center at 240 rpm by complex algebra analysis.

(VTU, Aug. 2000)

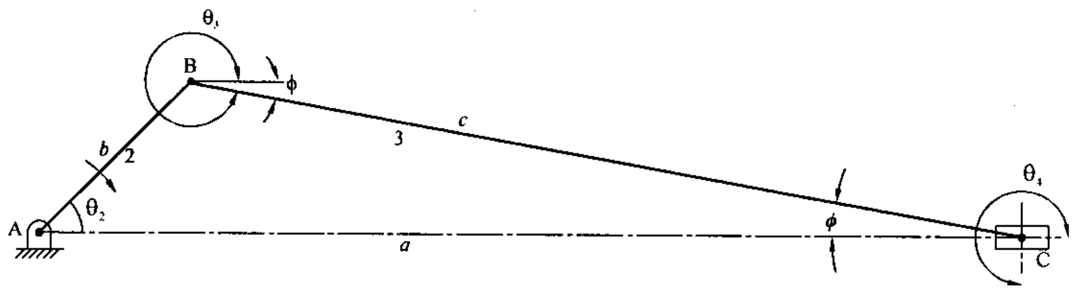

Fig. 5.10

$b = 200$ mm, $c = 4 \times 200 = 800$ mm, $\theta_2 = 45^\circ$, $n_2 = -150$ rpm (clockwise)

$\alpha_2 = 0$ (constant speed). Refer fig. 5.10

From triangle ABC

$$\frac{\sin\phi}{b} = \frac{\sin\theta_2}{c}$$

i.e., $$\frac{\sin\phi}{200} = \frac{\sin 45}{800}$$

$\therefore$ Angle $\phi = 10.182^\circ$

$\therefore$ Angle made by the link BC with horizontal $\theta_3 = 360 - 10.182 = 349.818$

Angular velocity of the crank $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times (-150)}{60} = -25.133$ rad/s

Angular velocity of connecting rod $\omega_3 = -\dfrac{b \cos \theta_2}{c \cos \theta_3} \times \omega_2$

$$= \frac{-200 \times \cos 45}{800 \times \cos 349.818} \times (-25.133)$$

$$= 4.514 \text{ rad/s}^2$$

Angular acceleration of the connecting rod.

$$\alpha_3 = \frac{\omega_3}{\omega_2} \alpha_2 + \frac{b\, \omega_2^2 \sin\theta_2 + c\, \omega_3^2 \sin\theta_3}{c \cos\theta_3}$$

$$= 0 + \frac{200 \times (-25.133)^2 \sin 45 + 800 \times 4.514^2 \sin 349.818}{800 \times \cos 349.818}$$

$$= 109.79 \text{ rad/s}^2$$

Real part of the slider acceleration equation is

$$R_a = -b\,(\omega_2^2 \cos \theta_2 + \alpha_2 \sin \theta_2) - c\,(\omega_3^2 \cos \theta_3 + \alpha_3 \sin \theta_3)$$

$$= -200\,[(-25.133)^2 \times \cos 45 + 0] - 800\,(4.514^2 \times \cos 349.818 + 109.79 \times \sin 349.818)$$

$$= -89848.99$$

Imaginary part of the slider acceleration equation is,

$$I_a = b\,(\alpha_2 \cos \theta_2 - \omega_2^2 \sin \theta_2) + c\,(\alpha_3 \cos \theta_3 - \omega_3^2 \sin \theta_3)$$

$$= 200\,[0 - (-25.133)^2 \times \sin 45) + 800\,(109.79 \cos 349.818 - 4.514^2 \times \sin 349.818) = 0$$

Acceleration of piston $A_p = \sqrt{R_a^2 + I_a^2}$

$$= \sqrt{(-89848.99)^2 + 0^2} = 89848.99 \text{ mm/s}^2$$

$$= 89.85 \text{ m/s}^2$$

Example 5.8

The crank radius of a reciprocating engine is 90 mm, the connecting rod is 360 mm long and the crank rotates at 150 rpm clockwise. By Raven's approach, determine the velocity and acceleration of the piston, and the angular velocity and angular acceleration of the connecting rod when the angle made by the crank with the inner dead center is 30°. Verify your answer by Klein's construction and hence find the percentage error in the graphical method.

Data :

$b = 90$ mm, $c = 360$ mm, $n_2 = -150$ rpm (clockwise), $\alpha_2 = 0$ (constant speed), $\theta_2 = 30°$

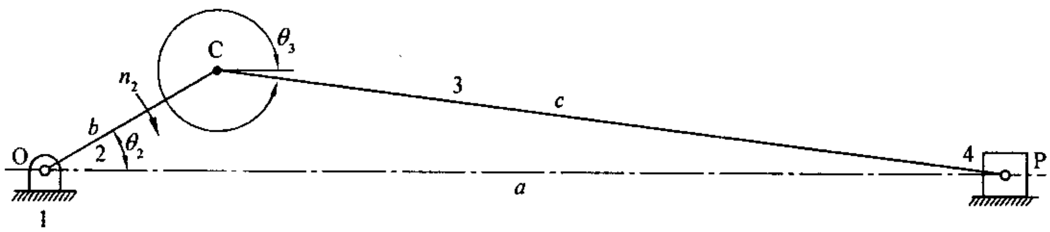

Fig. 5.11

Solution: *(refer fig. 5.11)*

Angle made by the connecting rod CP $\theta_3 = 360 - \sin^{-1}\left(\frac{b}{c}\ \sin\theta_2\right)$

$$= 360 - \sin^{-1}\left(\frac{90}{360} \times \sin 30\right) = 352.82°$$

Angular velocity of the crank OC, $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times (-150)}{60} = -15.7$ rad/s

Angular velocity of connecting rod CP, $\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$

$$= -\frac{90 \times \cos 30}{360\ \cos 352.82} \times (-15.7) = 3.426 \text{ rad/s}$$

Angular acceleration of the connecting rod CP, $\alpha_3 = \frac{b\omega_2^2\ \sin\theta_2 + c\omega_3^2\ \sin\theta_3}{c\ \cos\theta_3} + \frac{\omega_3}{\omega_2}\ \alpha_2$

$$= \frac{90 \times 15.7^2\ \sin 30 + 360 \times 3.426^2 \sin 352.82}{360\ \cos 352.82} = 29.576 \text{ rad/s}^2$$

Velocity of the piston:

Real part of velocity equation $R_v = -b\omega_2 \sin\theta_2 - c\omega_3 \sin\theta_3$

$= -90 \times (-15.7) \sin 30 - 360 \times 3.426 \sin 352.82 = 860.6$

Imaginary part $I_v = b\omega_2 \cos\theta_2 + c\omega_3 \cos\theta_3$

$= 0$ ($\therefore$ velocity vector is on the real axis)

$\therefore$ Velocity of piston $V_p = \sqrt{R_v^2 + I_v^2} = \sqrt{860.6^2 + 0} = 860.6$ mm/s $= 0.8606$ m/s

Direction $\theta_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\left(\frac{0}{860.6}\right) = 0°$

The velocity vector is away from the crank end.

Acceleration of the piston:

Real part of acceleration equation $R_a = -b(\omega_2^2 \cos\theta_2 + \alpha_2 \sin\theta_2) - c(\omega_3^2 \cos\theta_3 + \alpha_3 \sin\theta_3)$

$$= -90(15.7^2 \cos 30 + 0) - 360\,(3.426^2 \cos 352.82 + 29.576 \sin 352.82)$$

$$= -22073.6 \text{ mm/s}^2$$

Imaginary part $I_a = b(\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) + c\,(\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3)$

$= 0$ ($\because$ Acceleration vector is on the real axis)

$\therefore$ Acceleration of piston $A_p = \sqrt{R_a^2 + I_a^2} = \sqrt{22073.6^2 + 0^2} = 22073.6 \text{ mm/s}^2$

$$\theta_p = \tan^{-1}\left(\frac{0}{-22073.6}\right) = 180 - \tan^{-1}\left(\frac{0}{-22073.6}\right) = 180^\circ$$

($\because$ θ_p lies in 2nd quadrant)

$\therefore$ The acceleration vector is towards the crank end.

Klein's construction *(refer fig. 5.12)*

1. Draw the slider crank mechanism OCP for the given position of the crank with some suitable scale.
2. Produce the connecting rod PC to meet the vertical at O in M.
3. With PC as diameter draw a circle.
4. With C as center and radius equal to CM draw another circle to cut the previous circle at R and S.

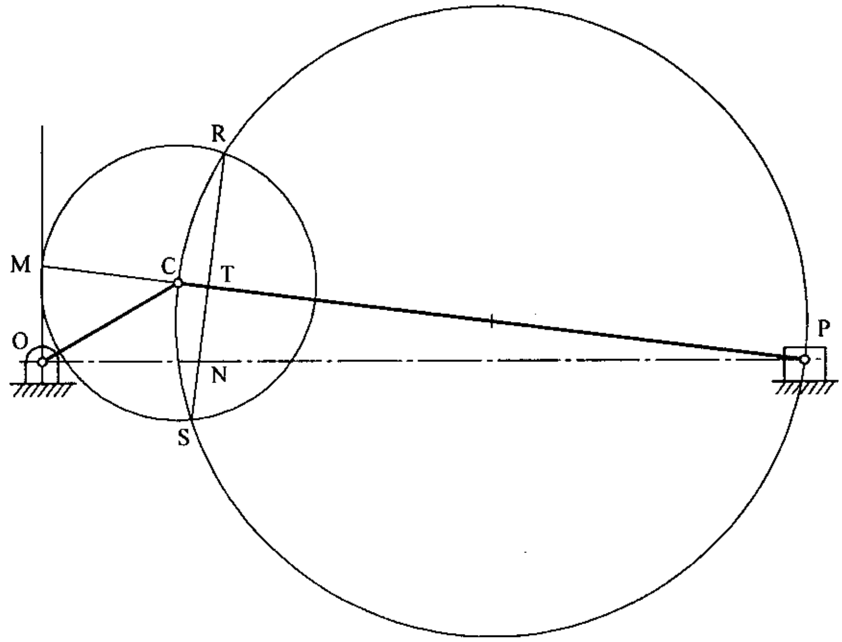

Fig. 5.12

5. Join SR cutting PC in T and OP in N.
   On measurement, the length OM = 54.87 mm, ON = 89.7 mm, CM = 78.66 mm and TN = 43.2 mm.

   Velocity of the piston $V_{p1} = \omega_2 \times OM = 15.7 \times 54.87 = 861.459$ mm/s

   Acceleration of the piston $A_{p1} = \omega_2^2 \times ON = 15.7^2 \times 89.7 = 22110.15$ mm/s²

   Angular velocity of the connecting rod

$$\omega_{31} = \omega_2 \times \frac{CM}{PC} = 15.7 \times \frac{78.66}{360} = 3.43 \text{ rad/s}$$

   Angular acceleration of the connecting rod

$$\alpha_{31} = \omega_2^2 \times \frac{TN}{PC} = 15.7^2 \times \frac{43.2}{360} = 29.58 \text{ rad/s}^2$$

$$\% \text{ error in velocity of piston} = \frac{V_p - V_{p1}}{V_p} \times 100 = \frac{860.6 - 861.459}{860.6} \times 100 = -0.0998\ \%$$

$$\% \text{ error in acceleration of piston} = \frac{A_p - A_{p1}}{A_p} \times 100$$

$$= \frac{22073.6 - 22110.15}{22073.6} \times 100 = -0.166\ \%$$

$$\% \text{ error in angular velocity of connecting rod} = \frac{\omega_3 - \omega_{31}}{\omega_3} \times 100$$

$$= \frac{3.426 - 3.43}{3.426} \times 100 = -0.117\ \%$$

$$\% \text{ error in angular acceleration of connecting rod} = \frac{\alpha_3 - \alpha_{31}}{\alpha_3} \times 100$$

$$= \frac{29.576 - 29.58}{29.576} \times 100 = -0.0135\ \%$$

Example 5.9

In an internal combustion engine mechanism, the crank radius is 100 mm and the length of the connecting rod is 450 mm. The crank is rotating at 10 rad/s in anti-clockwise direction. Determine the magnitude and direction of (i) Velocity of the piston and (ii) the angular velocity of the connecting rod when the crank is at 45° form the inner dead centre by complex algebra method. Verify the same by Klein's construction. **(VTU, Aug. 2001)**

Data:

$b = 100$ mm, $c = 450$ mm, $\omega_2 = 10$ rad/s (CCW), $\theta_2 = 45°$

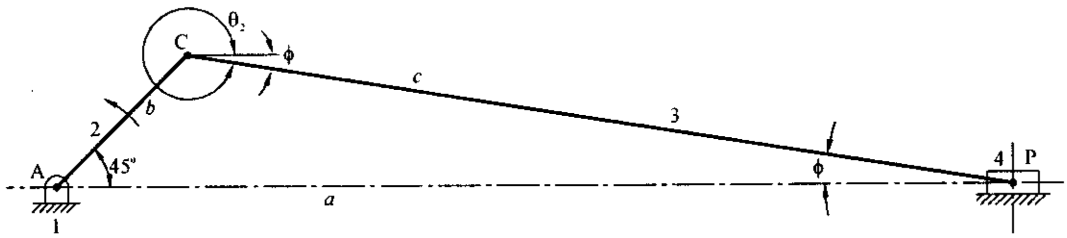

Fig. 5.13

Solution: *(refer fig. 5.13)*

Angle made by the connecting rod

$$\theta_3 = 360 - \tan^{-1}\left(\frac{b}{c}\sin\theta_2\right)$$

$$= 360 - \sin^{-1}\left(\frac{100}{450}\sin 45\right) = 350.96^{\circ}$$

Angular velocity of connecting rod $\omega_3 = \dfrac{-b\cos\theta_2}{c\cos\theta_3} \times \omega_2$

$$= \frac{-100\cos 45}{450\cos 350.96} \times 10 = -1.591 \text{ rad/s}$$

Angular acceleration of the connecting rod

$$\alpha_3 = \frac{\omega_3}{\omega_2}\alpha_2 + \frac{b\,\omega_2^2\sin\theta_2 + c\,\omega_3^2\sin\theta_3}{c\cos\theta_3}$$

$$= \frac{-1.591}{10} \times 0 + \frac{100 \times 10^2 \sin 45 + 450 \times (-1.591)^2 \sin 350.96}{450\cos 350.96}$$

$$= 15.508 \text{ rad/s}^2$$

Real part of the velocity equation $R_v = -b\,\omega_2\sin\theta_2 - c\,\omega_3\sin\theta_3$

$= -100 \times 10\sin 45 - 450 \times (-1.591)\sin 350.96 = -819.6$

Imaginary part of the velocity equation $I_v = b\,\omega_2\cos\theta_2 + c\,\omega_3\cos\theta_3$

$= 0$ ($\because$ velocity vector is on the real axis)

$\therefore$ Velocity of piston $V_p = \sqrt{R_v^2 + I_v^2} = \sqrt{819.6^2 + 0} = 819.6$ mm/s

Klein's construction: *(refer fig. 5.14)*

1. Draw the slider crank mechanism OCP for the given position of the crank with suitable scale.
2. Produce the connecting rod PC to meet the vertical at O in M.
3. With PC as diameter draw a circle

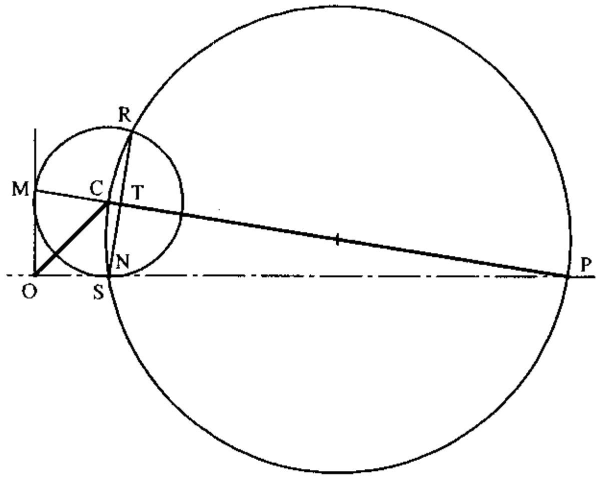

Fig. 5.14

4. With C as center and radius equal to CM draw another circle to cut the previous circle at R and S.
5. Join SR cutting PC at T and OP at N.

   On measurement, we get

   OM = 82 mm and CM = 70 mm

   Velocity of piston $V_p = \omega_2 \times OM = 10 \times 82 = 820$ mm/s

   Angular velocity of the connecting rod $\omega_3 = \omega_2 \times \dfrac{CM}{PC}$

   $$= 10 \times \frac{70}{450} = 1.56 \text{ rad/s}$$

Example 5.10

In a slider crank mechanism the direction of velocity and acceleration are shown in fig. 5.15. The crank radius is 0.1 m and the length of the connecting rod is 0.45 m. The crank makes an angle of 45° from the inner dead center and it rotates at constant speed. By complex algebra method, determine the angular velocity of the crank, angular velocity of the connecting rod, angular acceleration of the connecting rod and the acceleration of the slider if the velocity of the slider is 0.8196 m/s.

Data:

$b = 0.1$ m, $c = 0.45$ m, $\theta_2 = 45°$, $\alpha_2 = 0$ (constant crank speed),
$V_p = -0.8196$ m/s (direction of V_p is towards the crank end)

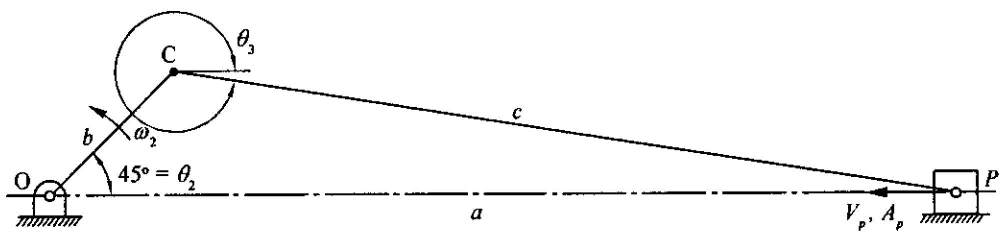

Fig. 5.15

Solution:

Angle made by the connecting rod with the horizontal (real) axis

$$\theta_3 = 360 - \sin^{-1}\left(\frac{b}{c}\sin\theta_2\right)$$

$$= 360 - \sin^{-1}\left(\frac{0.1}{0.45}\times\sin 45\right) = 350.96°$$

Real part of the velocity equation $R_v = -b\,\omega_2 \sin\theta_2 - c\,\omega_3 \sin\theta_3 = V_p$ $\quad(\because I_v = 0)$

i.e., $-0.1\,\omega_2 \sin 45 - 0.45\,\omega_3 \sin 350.96 = -0.8196$

or $-0.07071\,\omega_2 + 0.070706\,\omega_3 = -0.8196$ ..... (1)

Imaginary part of velocity equation $I_v = b\,\omega_2 \cos\theta_2 + c\,\omega_3 \cos\theta_3 = 0$

i.e., $0.1\,\omega_2 \cos 45 + 0.45\,\omega_3 \cos 350.96 = 0$

or $\omega_2 = -6.2849\,\omega_3$ ..... (2)

Substitute the value of ω_2 in terms of ω_3 in equation (1), we get

$-0.07071 \times (-6.2849\,\omega_3) + 0.070706\,\omega_3 = -0.8196$

$\therefore$ Angular velocity of connecting rod $\omega_3 = -1.591$ rad/s

Angular velocity of the crank $\omega_2 = -6.2849 \times (-1.591) = 10$ rad/s

Angular acceleration of connecting rod $\alpha_3 = \dfrac{b\omega_2^2 \sin\theta_2 + c\omega_3^2 \sin\theta_3}{c\cos\theta_3} + \dfrac{\omega_3}{\omega_2}\alpha_2$

$$= \frac{0.1\times 10^2 \sin 45 + 0.45\times(-1.591)^2 \sin 350.96}{0.45\times\cos 350.96} + 0 \quad (\because \alpha_2 = 0)$$

$\therefore \quad \alpha_3 = 15.508$ rad/s^2

Real part of acceleration of slider is

$$R_a = -b\,(\omega_2^2 \cos\theta_2 + \alpha_2 \sin\theta_2) - c\,(\omega_3^2 \cos\theta_3 + \alpha_3 \sin\theta_3)$$
$$= -0.1\,(10^2 \cos 45 + 0) - 0.45\,(1.591^2 \cos 350.96 + 15.508 \sin 350.96)$$
$$= -7.0995 \text{ m/s}^2$$

$\therefore$ Acceleration of slider $A_p = \sqrt{R_a^2 + I_a^2} = \sqrt{7.0995^2 + 0} = 7.0995 \text{ m/s}^2$

Example 5.11

The crank of an internal combustion engine is 50 mm long and the length of the connecting rod is four times the length of the crank. The mass center of the connecting rod is 80 mm from the crank pin. The crank rotates at a constant speed of 1000 rpm clockwise. By complex algebra, determine, (i) Angular velocity and angular acceleration of the connecting rod, (ii) Velocity and acceleration of the piston, (iii) Position of the piston, (iv) Velocity and acceleration of the mass center of the connecting rod, when the crank makes an angle of 30° with the inner dead center.

Solution:

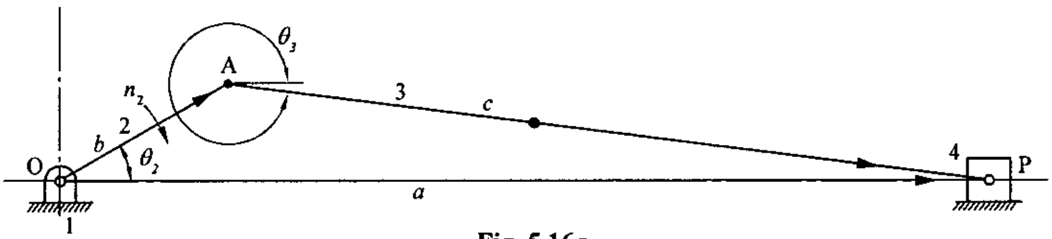

Fig. 5.16*a*

Data:

$b = 50$ mm, $c = 4 \times 50 = 200$ mm, $d = 80$ mm, $n_2 = -1000$ rpm (clockwise), $\theta_2 = 30°$

Solution:

Angle made by the connecting rod $\theta_3 = 360 - \sin^{-1}\left(\frac{b}{c}\sin\theta_2\right)$

$$= 360 - \sin^{-1}\left(\frac{50}{200} \times \sin 30\right) = 352.82°$$

Angular velocity of the crank $\omega_2 = \frac{2\pi n_2}{60} = \frac{2\pi \times (-1000)}{60} = -104.72$ rad/s

Angular velocity of the connecting rod $\omega_3 = -\frac{b\cos\theta_2}{c\cos\theta_3} \times \omega_2$

$$= \frac{-50 \times \cos 30}{200 \cos 352.82} \times (-104.72) = 22.852 \text{ rad/s}$$

Angular acceleration of the connecting rod

$$\alpha_3 = \frac{b\omega_2^2 \sin\theta_2 + c\omega_3^2 \sin\theta_3}{c\cos\theta_3} + \frac{\omega_3}{\omega_2}\alpha_2$$

$$= \frac{50\times(-104.72)^2 \sin 30 + 200\times 22.852^2 \sin 352.82}{200\cos 352.82} + 0$$

$$= 1315.833 \text{ rad/s}^2$$

Velocity of piston $V_p = -b\omega_2 \sin\theta_2 - c\omega_3 \sin\theta_3$ $(\because I_v = 0)$

$$= -50\times(-104.72)\sin 30 - 200\times 22.852 \sin 352.82$$

$$= 3189.24 \text{ mm/s} = 3.18924 \text{ m/s}$$

Acceleration of piston:

Real component of acceleration $R_a = -b(\omega_2^2\cos\theta_2 + \alpha_2\sin\theta_2) - c(\omega_3^2\cos\theta_3 + \alpha_3\sin\theta_3)$

$$= -50(-104.72^2\cos 30 + 0) - 200(22.852^2\cos 352.82 + 1315.833\sin 352.82)$$

$$= -545585.1844 \text{ mm/s}$$

Imaginary component $I_a = 0$ ($\therefore$ The acceleration vector is on the real axis)

Magnitude of acceleration $A_p = \sqrt{R_a^2 + I_a^2} = \sqrt{545585.1844^2 + 0}$

$$= 545585.1844 \text{ mm/s}^2 = 545.585 \text{ m/s}^2$$

Position of the slider $a = b\cos\theta_2 + c\cos\theta_3$

$$= 50\cos 30 + 200\cos 352.82 = 241.733 \text{ mm}$$

from the crank shaft center

Velocity and acceleration of mass center of the connecting rod Q:

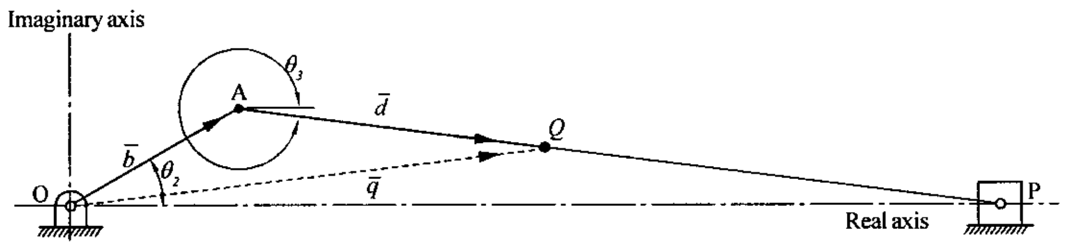

Fig. 5.16*b*

Mark the point Q on the connecting rod which lies at a distance of 80 mm from the crank pin. Join OQ. The loop - closure equation for the triangle OAQ is (refer fig. 5.16*b*)

$$\bar{q} = \bar{b} + \bar{d} \qquad \text{.....(1)}$$

Expressing the above equation in complex form,

$$q = be^{i\theta_2} + de^{i\theta_3} \quad \text{..... (2)}$$

Differentiating the equation (2) with respect to time, we get

Velocity of Q, $$V_Q = ib\frac{d\theta_2}{dt}e^{i\theta_2} + id\frac{d\theta_3}{dt}e^{i\theta_3} \quad \text{..... (3)}$$

$$= ib\,\omega_2 e^{i\theta_2} + id\,\omega_3 e^{i\theta_3} \qquad \left(\because \frac{d\theta}{dt} = \omega\right) \quad \text{..... (4)}$$

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in the above equation, we get

$$V_Q = ib\omega_2(\cos\theta_2 + i\sin\theta_2) + id\omega_3(\cos\theta_3 + i\sin\theta_3) \quad \text{..... (5)}$$

Real part of the equation (5) is

$$R_v = -b\omega_2\sin\theta_2 - d\omega_3\sin\theta_3$$

$$= -50\times(-104.72)\sin 30 - 80\times 22.852\sin 352.82 = 2846.5$$

Imaginary part of the equation (5) is

$$I_v = b\omega_2\cos\theta_2 + d\omega_3\cos\theta_3$$

$$= 50\times(-104.72)\cos 30 + 80\times 22.852\cos 352.82$$

$$= -2720.68$$

Velocity of Q, $$V_Q = \sqrt{R_v^2 + I_v^2}$$

$$= \sqrt{2846.5^2 + 2720.68^2} = 3937.6\text{ mm/s} = 3.9376\text{ m/s}$$

Direction of V_Q, $$\theta_v = \tan^{-1}\left(\frac{I_v}{R_v}\right) = \tan^{-1}\left(\frac{-2720.68}{2846.5}\right) \quad (\because \theta_v \text{ lies in the 4th quadrant})$$

$$= 360 - \tan^{-1}\left(\frac{2720.68}{2846.5}\right) = 316.3^\circ$$

The acceleration of Q is obtained by differentiating equation (3) with respect to time.

$$A_Q = b\,e^{i\theta_2}\left[i\frac{d^2\theta_2}{dt^2} - \left(\frac{d\theta_2}{dt}\right)^2\right] + d\,e^{i\theta_3}\left[i\frac{d^2\theta_3}{dt^2} - \left(\frac{d\theta_3}{dt}\right)^2\right]$$

$$= b\,e^{i\theta_2}\left(i\alpha_2 - \omega_2^2\right) + d\,e^{i\theta_3}\left(i\alpha_3 - \omega_3^2\right) \quad \left[\because \frac{d^2\theta}{dt^2} = \alpha\right] \quad \text{.....(6)}$$

Substitute $e^{i\theta} = (\cos\theta + i\sin\theta)$ in equation (6), we get

$$A_Q = b(\cos\theta_2 + i\sin\theta_2)(i\alpha_2 - \omega_2^2) + d(\cos\theta_3 + i\sin\theta_3)(i\alpha_3 - \omega_3^3) \text{..... (7)}$$

Real part of the equation (7) is

$$R_a = -b\,(\omega_2^2 \cos\theta_2 + \alpha_2 \sin\theta_2) - d\,(\omega_3^2 \cos\theta_3 + \alpha_3 \sin\theta_3)$$

$$= -50\,(104.72^2 \cos 30 + 0) - 80\,(22.852^2 \cos 352.82 + 1315.833 \sin 352.82)$$

$$= -503146.34 \text{ mm/s}^2$$

Imaginary part of the equation (7) is

$$I_a = b\,(\alpha_2 \cos\theta_2 - \omega_2^2 \sin\theta_2) - d\,(\alpha_3 \cos\theta_3 - \omega_3^2 \sin\theta_3)$$

$$= 50\,(0 - 104.72^2 \sin 30) - 80\,(1315.833 \cos 352.82 - 22.852^2 \sin 352.82)$$

$$= -383819.73 \text{ mm/s}^2$$

Acceleration of Q, $A_Q = \sqrt{R_a^2 + I_a^2} = \sqrt{503146.34^2 + 383819.73^2}$

$$= 632830.02 \text{ mm/s}^2 = 632.83 \text{ m/s}^2$$

Direction of A_Q, $\theta_A = \tan^{-1}\left(\dfrac{I_a}{R_a}\right) = \tan^{-1}\left(\dfrac{-383819.73}{-503146.34}\right)$ ($\because \theta_v$ lies in the 3rd quadrant)

$$= 180 + \tan^{-1}\left(\frac{383819.73}{503146.34}\right) = 217.34^\circ$$

EXERCISE – 5

1. Fig. 5.17 shows a four bar mechanism. The crank O_2B rotates at 10 rad/s anti-clockwise. By Raven's method, determine;
   (i) Angular velocity of the link BC and O_4C
   (ii) Angular acceleration of the link BC and O_4C
   (iii) Velocity and acceleration of the point C.

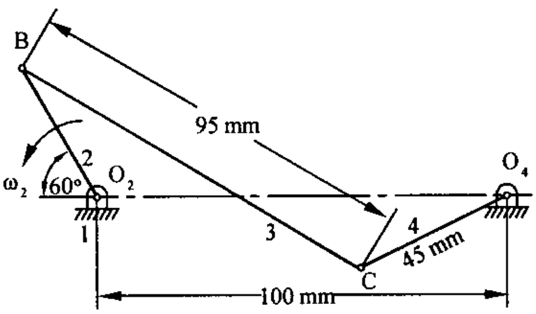

Fig. 5.17

2. The dimensions and data for a four bar mechanism shown in fig. 5.18 are given in the table. For each problem, determine θ_3, θ_4, ω_3, ω_4, α_3, and α_4 by using complex algebra. The angular velocity ω_2 is constant for each problem, and a negative sign is used to indicate the clockwise direction.

a	b	c	d	θ_2^o	ω_2 (rad/s)
250	100	500	400	70	–6
100	150	250	250	–45	56

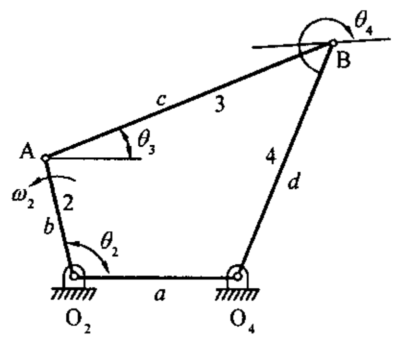

Fig. 5.18

[**Ans.** 28.3°, 235.9°, – 0.633 rad/s, – 2.16 rad/s, 7.82 rad/s^2, 6.7 rad/s^2]

[15.5°, 171°, 47.6 rad/s, 70.5 rad/s, 3330 rad/s^2, 3200 rad/s^2]

3. The crank of a reciprocating steam engine is 250 mm long and the length of the connecting rod is four times the length of the crank. The crank rotates at 180 rpm. The center of gravity of the connecting rod is 450 mm away from crank end. By complex algebra, determine;
   1. Velocity and acceleration of the piston.
   2. Angular velocity and angular acceleration of the connecting rod.
   3. Velocity and acceleration at the center of gravity of the connecting rod, for the crank position of (a) 30° and (b) 120° from the inner dead center position.

[**Ans.** 2.995 m/s, 89 m/s^2, 4.152 rad/s, 42.6 rad/s^2, 3.11 m/s, 85.4 m/s^2, and 3.54 m/s, 58.7 m/s^2, 2.6 rad/s, 80 rad/s^2, 4.05 m/s, 67.6 m/s^2]

4. Obtain loop closure equation for a four bar mechanism. Develop an equation for the relationship between the angular velocities of the input crank and the output crank of the four bar mechanism. **(VTU, July 2006)**

5. For the straight -line mechanism shown in fig. 5.19, ω_2 = 20 rad/s clockwise and α_2 = 140 rad/s^2 clockwise. By Raven's method determine the velocity and acceleration of point B and the angular acceleration of link 3. Take O_2A = AB = AC = 100 mm.

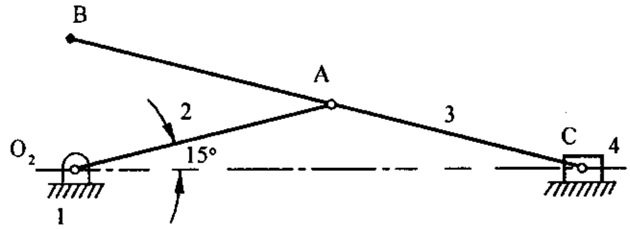

Fig. 5.19

6. The crank of an engine is 200 mm long and the ratio of connecting rod length to crank radius is 4. Determine the acceleration of the piston when the crank has turned through 45° from the inner dead center position and moving towards outer dead center at 240 rpm by complex algebra. [**Ans.** 89.85 m/s^2]

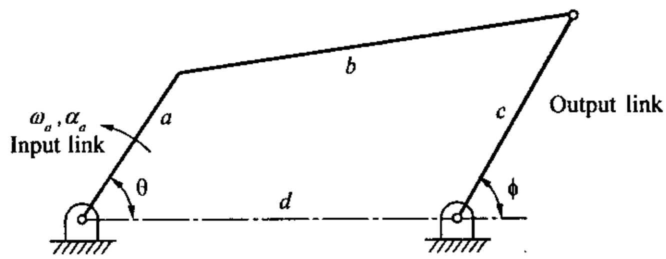

Fig. 5.20

7. A four bar mechanism is shown in fig. 5.20. Write the loop closure equation and determine the expression for (i) The output angle ϕ, (ii) The angular velocity of the output link.

(VTU, July 2005)